Identifying the mechanisms that generate choice and timing behavior in dynamic concurrent choice procedures

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Table of Contents

List of Figures

List of Tables

Abstract

IDENTIFYING THE MECHANISMS THAT GENERATE CHOICE AND TIMING BEHAVIOR IN DYNAMIC CONCURRENT CHOICE PROCEDURES

by

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Cognitive theories of timing and conditioned reinforcement provide two different theoretical perspectives on choice between delayed rewards. The primary objective of this research was to identify the process that generates choice in the concurrent-chains procedure and to characterize its relationship with temporal control. Experiments 1-3 investigated the relationship between the dynamics of pigeons' preference and temporal control in concurrent chains using an arrangement in which the delays to reinforcement changed unpredictably across sessions. To obtain convergent measures of choice and timing behavior, occasional 'no-food' terminal links lasted longer than the schedule values and ended without reinforcement. Measures of choice (log initial-link response ratios) and timing (start and stop times from no-food terminal links) stabilized within individual sessions. Sensitivity of log response ratios to relative immediacy increased as initial-link duration decreased or absolute terminal-link delays increased, but absolute initial- and terminal-link duration did not affect temporal control. Residual covariation analyses of log response ratios with log start and stop time ratios confirmed that measures of choice and timing were interdependent. Experiments 4 and 5 used concurrent-chains procedures in which immediacy, magnitude (and probability, in Experiment 5) ratios for left and right keys were 2:1 or 1:2, determined across sessions by independent, random series. Experiment 6 was a concurrent schedule in which relative reinforcement rate and

magnitude were 2:1 or 1:2, determined the same way. Multiple regression analyses showed that pigeons' response allocation in Experiments 4-6 was sensitive to multiple dimensions of reinforcement. Levels of preference within individual sessions and initial links or interfood intervals was more extreme when all dimensions favored the same key than when at least one dimension favored each key, consistent with assumptions of the generalized matching law. Within individual sessions, changes in response allocation in all experiments tended to be abrupt, consistent with the assumptions of Rate Estimation Theory (Gallistel & Gibbon, 2000). A decision model that posits a comparison between delayed outcomes with a criterion delay (Grace & McLean, 2006) described initial-link responding in Experiments 1-3. A modified decision model in which outcome expectancy is compared to an expectancy criterion described responding in Experiments 4-6.

1 Chapter 1

Introduction and literature review

The primary objective of this research was to attempt to identify the process that generates choice between delayed food rewards and to characterize its relationship with temporal control. Accounts of choice derived from cognitive theories of timing (Gallistel & Gibbon, 2000; Gibbon, 1977) assume a categorical or 'winner-take-all' process. By contrast, accounts of choice based on conditioned reinforcement (Fantino, 1969; Grace, 1994; Mazur, 2001) assume a direct correspondence between response allocation and relative value. These categorization and matching-to-value processes make similar predictions about stable behavior in unchanging conditions, but arrive at those predictions via different trajectories. In this thesis, I present research that addresses these issues using five concurrent-chains and one concurrent schedule experiment in which the contingencies placed on reinforcement change unpredictably each session.

The first chapter contains a selective review of extant empirical and theoretical literature about the process that generates choice between delayed rewards and other published material relevant to the methodology used in the present research. It begins with an overview of matching, determiners of choice in concurrent chains and a description of three 'steady-state' models of choice between delayed rewards. In steady-state operant research, contingencies do not change within conditions and it is stable performance that is analyzed and explained. Theoretical and empirical work on choice in transition is considered separately. To provide the necessary background for analyses of temporal control in this thesis, Chapter 1 summarizes steady-state and dynamic research on interval timing and theories of temporal control as they apply to concurrent chains. Six experiments

follow which used methods of converging operations (Garner, Hake & Eriksen, 1953) to obtain measures of choice and timing under dynamic conditions. The decision model (Grace & McLean, 2006) is a quasidynamic model for choice between delayed rewards. It may be possible to extend the decision model to incorporate the relationship between response allocation and temporal control identified in Experiments 1-3 and to account for effects of other reinforcer dimensions on response allocation in Experiments 4-6. This possibility is addressed in the General Discussion.

1.1 Matching and the empirical study of choice

With a series of concurrent schedule and concurrent chain experiments, Herrnstein (1961, 1964) initiated a research paradigm investigating the principle of 'matching' that has inspired and informed the study of free-operant choice ever since (see also Herrnstein, 1970). The core principle of matching is that preference for a particular alternative, operationally defined as relative behavior or time allocation, equals or *matches* the 'value' of that alternative in relation to whatever other alternatives may be available. Many steady-state and some dynamic choice experiments provide empirical support for the assumption that matching drives the underlying mechanism producing preference behavior.

1.1.1 Strict matching to rate of reinforcement

Herrnstein (1961) trained pigeons in a concurrent-schedules procedure in which two independent variable-interval (VI) schedules operated on separate keys. Under a VI schedule of reinforcement, the first response after a specified interval has elapsed produces reinforcement, typically food. The amount of time that elapses before a response produces food varies from reinforcer to reinforcer. Throughout Herrnstein's experiment, one reinforcer was scheduled every 90 s on average. Across conditions, VI schedule values

ranged from 90 to 180 s for the richer key and 90 s to ∞ (extinction) for the leaner key. For conditions in which a 1.5 s changeover delay penalized rapid rates of switching and strict alternation (which had been reinforced during pretraining), Herrnstein reported that the proportion of pecking responses made to a key equaled the proportion of reinforcers received following a peck to that key:

$$\left(\frac{B_L}{B_L + B_R}\right) = \left(\frac{R_L}{R_L + R_R}\right).$$

Equation 1.1

In Equation 1.1, *B* variables are response rates and *R* variables are reinforcement rates. Subscripts *L* and *R* refer to left and right alternatives, respectively. In Herrnstein's experiment, relative current contingencies of reinforcement determined relative responding.

1.1.2 Strict matching to relative immediacy and other reinforcer dimensions

After Herrnstein's (1961) concurrent schedule experiment had established the matching relation between preference, or relative responding, and relative rate of reinforcement, investigations of how other types of reinforcer contingencies affected preference soon followed. Catania (1963) manipulated the magnitude of reinforcement by varying the duration that subjects (3 pigeons) could access food via grain hopper presentation from 3-6 s. Reinforcer magnitude had no effect on response rate in single VI schedules. However, when different reinforcer magnitudes were arranged on concurrent VI 120 VI 120 s schedules, response rate to a key was a linear function of duration of food presentation; therefore, the ratio of left to right responses matched the ratio of left reinforcer magnitude to right reinforcer magnitude.

INTRODUCTION

Herrnstein (1964) applied the mathematical principles observed in the relationship between relative responding and relative reinforcement (Herrnstein, 1961) to study the effectiveness of secondary reinforcement quantitatively. In a concurrent-chains procedure with VI 60-s initial links associated with two variable ratio (VR) or one VR and one VI terminal link, the relative initial-link response rates approximated relative rates of primary reinforcement in terminal links. Considering these results together with the matching of relative responding to relative primary reinforcement rate observed in concurrent schedules (Herrnstein, 1961), Herrnstein (1964) concluded that manipulating the frequency of secondary reinforcement was equivalent to manipulating the frequency of primary reinforcement.

For a quantitative assessment of choice between delayed rewards, Chung and Herrnstein (1967) reinforced pigeons in concurrent chains with consisting of two independently-scheduled VI 60-s initial links and fixed-time (FT) terminal links, in which reinforcement is not contingent on any terminal-link response but it does not occur until a scheduled delay has elapsed since terminal-link entry. Whereas the terminal-link schedule for one key was always FT 8 s, the schedule for the other terminal link was varied from FT 1 s to FT 30 s across conditions. The relationship between initial-link response proportions and terminal-link delay proportions was negative and approximately linear. Because longer delays are aversive to hungry subjects, Chung and Herrnstein expressed the matching relation for this experiment in terms of relative immediacy, the reciprocal of delay:

$$\frac{B_L}{B_R} = \frac{1/D_L}{1/D_R}$$

Equation 1.2

In Equation 1.2, *D* represents terminal-link delay to reinforcement and all other variables and subscripts are as in Equation 1.1. Chung and Herrnstein found that relative responding matched relative terminal-link immediacy.

Schneider (1968) reinforced pigeons according to concurrent-chains schedules with VI initial and terminal links. He reported that when terminal links ended with reinforcement on either 100% or 50% of trials, initial-link response ratios matched relative rate of reinforcement on terminal links, as determined by both immediacy and probability of reinforcement. Thus, early research on choice indicated that relative rates of responding matched relative primary and secondary rates of reinforcement.

1.1.3 Generalized matching

A strict interpretation of the matching law as outlined by Herrnstein (1961) and presented in Equations 1.1 and 1.2 is that relative response (or time) allocation should equal the contingency ratio exactly. The idealized functional relationship between response ratio (B_L/B_R) and reinforcer contingency (e.g. rate, magnitude or immediacy) ratio is linear with a slope of one. Any systematic deviations in linearity or slope constitute violations of this "strict" matching relation. However, a generalization of Herrnstein's equation can account for deviations in linearity or slope. The generalized matching law (Baum, 1974; 1979; McCarthy & Davison, 1988) states that relative responding is a power function of the reinforcer ratio:

$$\frac{B_L}{B_R} = b\left(\frac{X_L}{X_R}\right)^a$$

Equation 1.3

In Equation 1.3, X is the manipulated reinforcer dimension, b a constant preference favoring the left or right alternative respectively if greater or less than 1, a represents response-allocation sensitivity to dimension X. Dimension X can refer to reinforcer rate, magnitude, immediacy, probability or any other dimension of phylogenetic or conditioned importance. Equation 1.3 is a mathematical expression of the idea that response allocation between two (or more) alternatives matches their relative value. The determination of the value of an alternative may involve linear or nonlinear transformations of objective reinforcer values such as rate, immediacy, magnitude and probability.

The idea that relative responding matches relative value can also be expressed in terms of proportions (i.e., as a generalization of Equation 1.1) and it is frequently expressed in terms of log ratios:

$$\log\left(\frac{B_L}{B_R}\right) = \log b + a\log\left(\frac{X_L}{X_R}\right)$$

Equation 1.4

According to the strict matching law, the relationship between log response ratio and log reinforcer ratio is a straight line with a slope (sensitivity, a) of one and intercept (bias, log$b$) of zero.

Figure 1.1 shows relative responding as a function of relative reinforcer contingency, expressed as proportions, ratios and log ratios. Panels A-C show idealized functions with no bias (b=1) and sensitivity ranging from 0.5 to 2. Panels D-F show idealized functions with strict matching sensitivity (a=1) and bias ranging from 0.5 to 2. The yellow lines in all panels of Figure 1.1 represent strict matching of relative responding to relative reinforcer contingencies.

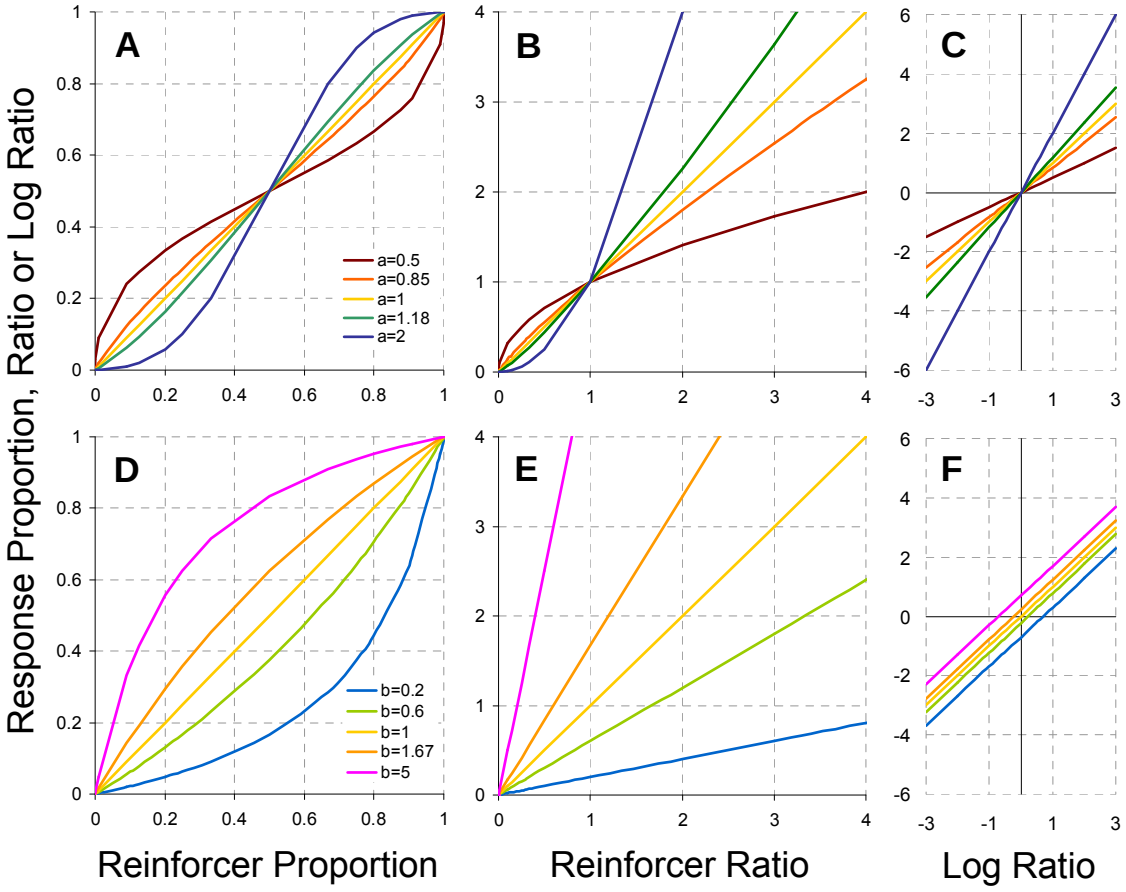

Figure 1.1. Sensitivity and bias.

Baum (1974; 1979) described situations in which subjects' performance typically differed from strict matching. He reported that in many concurrent-schedules experiments, response ratios were less extreme (closer to indifference, which would be 1:1) than reinforcement ratios. Fantino, Squires, Delbrück and Peterson (1972) referred to this lower-than-predicted by strict matching sensitivity to relative reinforcement as *undermatching*. By contrast, Omino and Ito (1993) exposed pigeons to concurrent chains with fixed interval (FI) terminal links in which time from terminal-link onset to reinforcer availability did not vary. They reported that *overmatching*, in which response ratios were more extreme than immediacy ratios, predominated. In the top panels of Figure 1.1 (A-C), the red and orange lines (a = 0.5 and a = 0.85) represent examples of undermatching and the green and blue lines (a = 1.18 and a = 2) of overmatching.

Baum (1974) also noted the importance of distinguishing the differences in sensitivity to reinforcement that describe under- and overmatching from bias, a constant preference for one alternative over another that is independent of experimentally manipulated reinforcer contingencies. Bias in favor of a particular alternative is generally assumed to be due to factors affecting reinforcer (or conditioned reinforcement) value that are not under experimental control. Researchers have exploited the idea of 'bias' to investigate choice between qualitatively different reinforcers. Hollard and Davison (1971) presented pigeons with two-key VI-VI concurrent schedules in which responses to one key were reinforced with food and responses to the other with brain stimulation. Across five conditions, the schedule associated with the food key varied from VI 30 s to VI 630 s. The schedule associated with the brain stimulation key was always VI 60 s. Hollard and Davison applied Equation 1.4 for primary reinforcement rate to log response ratios (food key/brain stimulation key) from the last 5 experimental sessions of each condition. For all three subjects, log$b$ was positive, indicating a constant preference for the food key over the brain stimulation key. Similarly, Miller (1976) employed different pairs of independent VI schedules in a two-key concurrent-schedules procedure to establish pigeons' transitive preference for buckwheat over hemp and wheat over buckwheat.

The matching relation can be expressed in terms of response and reinforcer proportions (panels A and D in Figure 1.1), ratios (B and E), and log ratios (C and F; Equation 1.4). However, Baum (1974) noted that the three expressions are not equally suited to evaluating sensitivity and bias in choice between reinforcement alternatives. So long as subjects exclusively prefer a reinforced alternative to extinction, strict matching is necessarily the best-fitting line when response proportions are regressed linearly on reinforcer proportions. Manipulating relative reinforcement across conditions constrains

the number of proportions it is possible to test, which can impede distinguishing between residual error and systematic deviations from matching such as bias, undermatching or overmatching. Determining whether strict matching applies to ratios can be more straightforward. However, experimentally distinguishing the exponential functions associated with under- and overmatching from the non-identity linear functions associated with bias requires comparatively sophisticated statistical analyses.

By contrast, when log response ratios are regressed linearly on log reinforcer contingency ratios, slope and intercept respectively reflect sensitivity and bias. Over- or undermatching has occurred if the slope parameter (a in Equation 1.4) is meaningfully greater or less than one. A nonzero intercept ($\log b$) reflects a bias. Because differences in sensitivity and bias are easily identified using linear regression, even given a limited number of conditions, Equation 1.4 is a stronger, more straightforward way to describe and evaluate matching of relative responding to relative reinforcer contingencies. In the case of choice between delayed rewards:

$$\log\left(\frac{B_L}{B_R}\right) = \log b + a\log\left(\frac{1/D_L}{1/D_R}\right)$$

Equation 1.5

According to Equation 1.5, log response ratio is a linear function of log immediacy ratio. Although exclusive preference and extinction cannot be appropriately represented using log ratios, Equation 1.5 is the best way to describe and evaluate sensitivity to immediacy in dependently-scheduled concurrent chains in which both alternatives are reinforced. Contemporary generalized-matching analyses are typically reported in log ratio format.

1.1.4 Concatenated matching

When researchers are interested in more than one reinforcer dimension, the question of how different dimensions combine to affect relative responding becomes relevant. Baum and Rachlin (1969) suggested that the simplest possible way multiple reinforcer dimensions could combine to determine value was multiplicative:

$$\frac{B_L}{B_R} = \prod_{i=1}^{n} \frac{X_{iL}}{X_{iR}} = \frac{V_L}{V_R}.$$

Equation 1.6

Equation 1.6 states that the matching law applies to the relative value associated with each alternative, which is determined by a multiplicative concatenation of reinforcer contingency ratios. Variables V_L and V_R are the overall values of left and right alternatives, respectively. When log-transformed and generalized to account for bias and over- or undermatching between responding and individual dimensions (Killeen, 1972), Equation 1.6 becomes:

$$\log\left(\frac{B_L}{B_R}\right) = \log b + \left[\sum_{i=1}^{n} a_i \log\left(\frac{X_{iL}}{X_{iR}}\right)\right].$$

Equation 1.7

Equation 1.7 is known as the concatenated generalized matching law (Davison, 1983; Davison & McCarthy, 1988). According to Equation 1.7, an additive, independent weighted concatenation of the log ratios of different reinforcer dimensions determines log response allocation. Concatenated generalized matching is a testable empirical hypothesis because it states that the effects of different reinforcer dimensions are linearly independent.

Table 1.1. Free-operant choice experiments in which multiple reinforcer ratios were manipulated.

Study[1]	Independence[2]	Data Type[3]	Procedure[4]	Species[5]
Rate and magnitude in concurrent VI-VI schedules or concurrent chains[6]				
Schneider (1973)		R	DEP	P
Todorov (1973)		R	COK	P
Hamblin & Miller (1977)		R	IND	R
Davison (1988)	N	R, T	DEP	P
Leon & Gallistel (1998)	Y	T	IND	R
McLean & Blampied (2001)	Y	R	IND	P
Grace, Bedell & Nevin (2002)	Y	R	DEP[7]	P
Rate and immediacy in concurrent chains				
Davison (1976)	N	R	IND	P
Davison (1983)	Y	R, T	DEP	P
Berg & Grace (2004)	Y	R	DEP[7]	P
Immediacy and magnitude in concurrent chains				
Rodriguez & Logue (1986)	Y	R	DEP	P
Logue, Forzano & Tobin (1992)	Y	R	DEP	H
Grace (1995)	Y	R, T	DEP[7]	P
Immediacy and probability in concurrent chains				
Mattson (*unpublished doctoral thesis*), Experiment 4	Y	R	BOTH[7,8]	P
Magnitude and probability in concurrent chains				
Mattson (*unpublished doctoral thesis*), Experiment 5	Y	R	BOTH[7,8]	P
Rate and force in concurrent VI-VI schedules				
Hunter & Davison (1982)	Y	R, T	DEP	P
Rate and hedonic quality in concurrent VI-VI schedules				
Hollard & Davison (1971)		R, T	DEP	P

[1] Studies that manipulated absolute but not relative values, manipulated different dimensions across different experiments or involved single-response procedures are not included.
[2] Whether the assumption of dimensional independence (reinforcer dimensions have independent effects on choice) is supported (Y) or contradicted (N), if it was assessed.
[3] R and T indicate responses and time allocation as dependent variables.
[4] Whether the procedure used a changeover key (COK), or whether alternatives were scheduled dependently (DEP) or independently (IND) on two keys.
[5] Subjects were pigeons (P), rats (R), or humans (H).
[6] All experiments except Grace, Bedell & Nevin (2002) are concurrent chains
[7] Experiment involved a 3-component concurrent-chains procedure
[8] Initial links were scheduled dependently for half of subjects and independently for the other half

Table 1.1 summarizes empirical free-operant studies in which multiple reinforcer ratios were manipulated across conditions. No previously published concurrent-schedule or

concurrent-chains experiments have reported effects of varying more than two dimensions. Subjects in all 18 studies were sensitive to both varied reinforcer dimensions, supporting the concatenated-matching assumption of (logarithmic) additivity.

An additional assumption of the concatenated generalized matching law formalized in Equation 1.7 is that the effects of different reinforcer dimensions are independent. This assumption holds if there are no interactions between different reinforcer dimensions. Of the 13 studies in Table 1.1 reporting sufficient information to determine whether effects of dimensions were independent in log form, only two studies reported interactions.

Davison (1976) trained pigeons in concurrent chains in which the terminal-link immediacy ratio was 3:1, 1:1 or 1:3 and relative terminal-link entry rates ranged from 27:115 to 115:27. Pigeons' initial-link response allocation was sensitive to both entry rate and immediacy ratios. Sensitivity to rate was lower in conditions in which rate favored one key and immediacy the other than when one terminal link was both shorter and occurred more frequently; in the first situation, responding was less extreme than predicted by independent effects of reinforcer rate and immediacy. However, using similar procedures, Davison (1983), and Berg and Grace (2004) found no evidence of an interaction between rate and immediacy and concluded that Equation 1.7 provided an adequate and complete description of the combined effects of varying reinforcer rate and immediacy ratios in concurrent chains.

Davison (1988) reported that the change in sensitivity to rate of reinforcement as overall time to reinforcement (determined by two-key dependent concurrent VI schedule values) increased depending on relative reinforcer duration (or magnitude). Results of

some other studies indicate that response (Grace, Bedell & Nevin, 2002; McLean & Blampied, 2001) and time (Leon & Gallistel, 1998) allocation are determined by independent concatenations of reinforcer rate and magnitude (c.f. Elliffe, Davison & Landon, 2008). These other results suggest Davison's (1988) result may be more related to temporal context effects (discussed in Section 1.2.5) than a violation of the concatenated generalized matching law assumption that effects of log reinforcer contingency ratios on choice are linear.

Table 1.1 does not include any of the many experiments in which subjects made a single response each trial. However, results of discrete-trial (see Logue, 1988 for a review) and adjusting-delay (Mazur, 1987) experiments involving multiple dimensions are typically consistent with concatenated matching. Taken together, choice research using a variety of methodological approaches suggests that the concatenated matching law (Equation 1.7) provides a quantitative account of choice between alternatives that differ on two dimensions that is accurate to a first approximation. Further, results support the assumptions that log reinforcer ratios combine in a linearly independent fashion to determine log response ratios. Empirical results from steady-state experiments support the matching-law assumption of a mapping of value onto behavior.

1.1.5 Determinism, radical behaviorism and 'choice'

Behaviorism, particularly Skinnerian radical behaviorism (Skinner, 1971), provides the philosophical and doctrinal foundation for the experimental analysis of behavior developed by Skinner primarily (Ferster & Skinner, 1957), and used by Herrnstein and others interested in the empirical study of choice. From a behaviorist perspective, the goal of psychology is to describe and predict behavior as a function of environmental

contingencies and previous experience. For behaviorists, psychology is the science of behavior. This position opposes other theoretical approaches that view psychology as the study of mind. Behaviorism is a deterministic philosophy in that behavior is understood to be determined by environmental contingencies and previous experience, rather than unobservable, inaccessible internal mental states (Baum, 2005). By contrast, the lay notion of a 'choice' is an exercise in free will. Defined this way, choice is a problematic topic of study for behaviorists. The free-operant conceptualization of choice distinguishes Herrnstein's studies from other psychological approaches to choice and decision making and reconciles choice with the deterministic nature of behaviorism.

Skinner mentions choice and matching in particular in the (1950) monograph in which he questions whether 'theories of learning [are] necessary.' Skinner clarified that no explanatory description or prediction of data is wholly free of "certain basic assumptions" (p. 193) but classified as problematic the terms "*preferring, choosing, discriminating* and *matching*" because they were traditionally used as referring to "processes in another dimensional system." (p. 210). Skinner argued that empirical scientific knowledge can and usually does accumulate without a theory to guide it; therefore, theories of learning represent "a refuge from the data" (p. 195) and are inconsistent with a scientific approach to the study of behavior.

The four terms Skinner (1950) identified as traditionally theoretical did not appear in any of the empirical papers first- or sole-authored by Herrnstein prior to 1961. Herrnstein's (1961) *Relative and absolute strength of response as a function of frequency of reinforcement* is widely cited as the inaugural paper on the free-operant study of choice. However, the words 'choice' and 'preference' do not appear, and while Herrnstein used the

word ‘matching’ frequently, it referred to the observed equality between relative responding and reinforcement rather than an appeal to the unobservable (i.e., subjective value). Today, experimental behavior analysts use the words ‘choice’ and ‘preference’ in a manner reconcilable with determinism and Skinner’s (1950, p. 215) view that there was “no *a priori* reason why a complete account [of matching] is not possible without appeal to theoretical processes in other dimensional systems.”

Free-operant choice and preference are not expressions of free will, but rather relative frequencies of behaviors, shaped by reinforcement history and evolution (that is, by the contingencies the individual experienced since birth and by those contingencies that determined the reproductive fitness of its ancestors, respectively). Choice can be considered a selection of one out of two or more possible courses of action, and thus conceptualized the study of choice is consistent with the determinist framework of behaviorism. Operational definitions of this deterministic variety of choice amount to measures of allocation. In discrete-trials procedures, a single response constitutes a choice. In free-operant studies of choice, traditional measures include response allocation (Herrnstein, 1961), formulated as a proportion ($B_L/(B_L+B_R)$) or a ratio (B_L/B_R), and relative time allocation (Baum & Rachlin, 1969). Conceived this way, nearly all behaviors are choices. Even single-key experiments, including simple schedules, require subjects to choose whether to respond or not (Herrnstein, 1970). When derived from responding to individual operants, it is possible to quantify and model choice in simple and complex schedules of reinforcement, without appealing to theory.

1.2 Concurrent chains

Experimental behavior analysts have used the concurrent chains procedure introduced by Autor (1960, 1969) to study choice between delayed rewards for more than four decades. These original experiments combined the chain and concurrent schedules described by Ferster and Skinner (1957). A chained schedule of reinforcement is a situation in which two or more schedules operate consecutively. One stimulus signals the operation of the first schedule, or *initial link*. Instead of primary reinforcement, the response that satisfies the requirements of the first schedule produces a second schedule, signaled by a different stimulus. Only the final schedule in the chain, the *terminal link*, ends with reinforcer delivery. A concurrent schedule is a situation in which two or more schedules operate simultaneously. Each schedule requires its own stimulus, manipulandum and response. With pigeons, each schedule typically operates on a separate keylight. Pecks to a particular key can only satisfy the schedule operating on that key.

Autor programmed separate two-link chain schedules to operate on different keys in a two-key operant chamber. In his experiments, VI 60-s initial links operated concurrently. Upon entry into one terminal link, the VI schedule operating on the other key was paused; the key darkened and responses to it had no effect. Terminal links were pairs of VI, VR, or differential reinforcement of other behavior schedules. The terminal-link schedule value was fixed for one key and varied across conditions for the other. Autor measured response rates in initial links and found that relative initial-link response rates were approximately equal to relative rates of reinforcement in terminal links.

1.2.1 Contemporary concurrent chains procedure

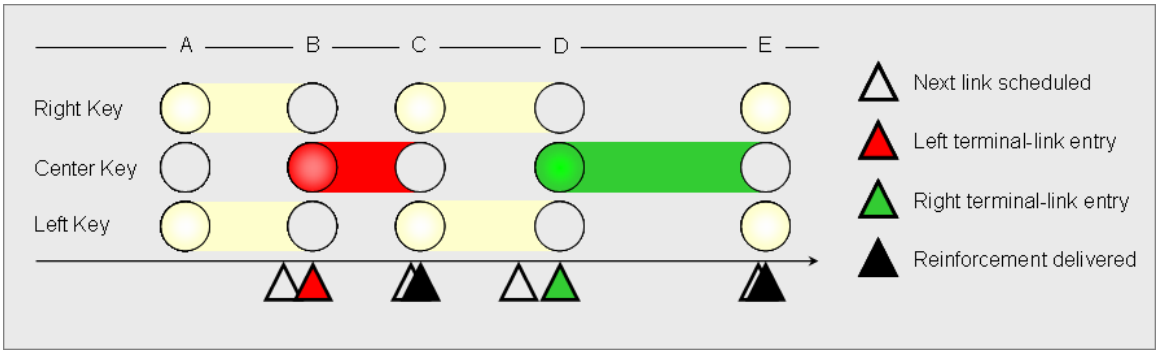

Figure 1.2. Concurrent-chains procedure.

Figure 1.2 is a diagram of a contemporary concurrent-chains procedure used with pigeons to study choice between delayed rewards. The horizontal axis represents time. Three circles represent keylights in a standard three-key operant chamber. Open triangles indicate a terminal-link entry or reinforcer becoming available pending a response. Red, green, and black triangles respectively indicate when a left or right terminal-link entry or reinforcer delivery occurs. Letters denote stimulus changes.

Line segments AB and CD represent initial links. During initial links, the side keys are lighted white and a single VI schedule operates. A pseudorandomly preselected half of the initial links in an experimental session end with a left terminal-link entry following the first peck to the left key that satisfies a changeover delay and occurs after the interval operating in the initial link has elapsed. The remaining initial links end with the first right terminal-link entry that satisfies the same criteria.

Upon terminal-link entry, side keys darken and the center key is lighted red (for left terminal links, line segment BC) or green (right, segment DE). A different FI schedule operates in each terminal link; once the scheduled interval elapses, the first center-key response is reinforced with food. Initial links are reinstated after reinforcer delivery.

1.2.2 Procedural considerations in initial-link scheduling

There are a few important procedural differences between the initial links described in section 1.2.1 and those employed by Autor (1960, 1969). In Autor's experiments, a separate VI schedule operated independently for each initial link. With independent initial links, a feedback relationship between preference and terminal-link experience always occurs: obtained relative terminal-link entry rates are determined by the subject's behavior as well as by programmed relative entry rates. To circumvent this problem, Stubbs and Pliskoff (1969) employed a single VI schedule that applied to both alternatives of a concurrent schedule. On each trial, separate circuits determined interval duration and which of two alternatives would produce reinforcement. Once the interval timed out, a response to the preselected alternative produced reinforcement, but a response to the other alternative had no effect. Stubbs and Pliskoff controlled the relative reinforcement rate obtained in concurrent schedules. In concurrent-chains procedures, dependently scheduled initial links (described in Section 1.2.1) control relative terminal-link entry rate. How does performance in independently- and dependently-scheduled initial links compare? Davison (1983) manipulated initial- and terminal-link duration over 61 conditions in each of two concurrent-chains experiments. Obtained terminal-link entry rates deviated from programmed rates more with independent than dependent initial links, however, Davison reported no consistent difference in pigeons' preference.

Ferster and Skinner (1957, p. 734) defined the VI schedule as "a schedule of intermittent reinforcement in which reinforcements are programmed according to a random series of intervals having a given mean and lying between arbitrary extreme values." In practice, experimenters typically sample from a finite series of intervals that approximates randomness (i.e., a geometric progression; Fleshler & Hoffman, 1962). However, other

types of progressions have been used. Catania and Reynolds (1968) compared pigeons' performance on single VI schedules drawn from different types of distributions. They found that response rate plotted as a function of time since reinforcement depended on the distribution that produced the intervals, consistent with sensitivity to conditional probability of reinforcement. A reanalysis of 18 experiments using concurrent VI VI schedules (Taylor & Davison, 1983) and a within-subjects comparison of performance under arithmetic and exponential concurrent schedules (Elliffe & Alsop, 1996) both found that response allocation was less sensitive to reinforcement rate when VI schedules were arithmetic than when they were exponential.

Most concurrent schedules and concurrent-chains procedures include a changeover delay (COD), a requirement that responses have occurred on a particular alternative for a minimum amount of time before the next response can produce a reinforcer (or terminal-link entry). CODs prevent reinforcement from occurring with high rates of switching between alternatives. The relationship between COD duration and sensitivity to reinforcement rate in concurrent schedules (Brownstein & Pliskoff, 1968; Davison, 1991), or to terminal-link immediacy in concurrent chains (Baum, 1974; Silberberg, Hamilton, Ziriax& Casey, 1978) is negative: rates of changing over from one alternative to another decrease as COD increases.

Several details relating to the initial links themselves, including dependent or independent scheduling, interval distribution shape and COD duration, can determine rate and topography of responding in initial links. However, the canonical finding of concurrent-chains research, matching to relative terminal-link immediacy, has occurred in concurrent chains with independently and dependently scheduled initial links of different

distributions and many nonzero COD durations. It remains robust after more than four decades of concurrent-chains choice experimentation.

1.2.3 The initial-link effect

Because relative terminal-link immediacy is the usual dimension of interest in concurrent-chains experiments, programmed initial-link schedules are typically identical for all alternatives and conditions within an experiment. To assess effects of absolute conditioned reinforcer rate on choice between delayed rewards, Fantino (1969) presented pigeons with concurrent chains in which terminal-link entries were arranged according to independent, equal VI schedules. Across conditions, initial-link VI schedule values were short, intermediate and long, 40, 120 and 600 s. Each subject experienced the conditions in a different order. Terminal links were reinforced with food according to VI 30-s and 90-s schedules. The initial-link key that produced the richer terminal link alternated across conditions. For all 6 subjects, preference for the VI 30-s terminal link was greatest in the short initial-link condition and closest to indifference in the long initial-link condition. Fantino concluded that the matching relation does not adequately describe choice between conditioned reinforcers. Other researchers have replicated the effect that initial-link duration attenuates preference with pigeons (Alsop & Davison, 1988; Berg & Grace 2006; Fantino & Royalty, 1987; Grace, Berg & Kyonka 2006; Mazur, 2004) and humans (Belke, Pierce & Powell, 1989).

1.2.4 The terminal-link effect

Whereas longer initial links attenuate preference in concurrent chains, longer absolute terminal-link delays magnify sensitivity to immediacy ratio. MacEwen (1972) trained pigeons in concurrent chains with FI terminal links. Terminal-link immediacy

ratios were always 2:1 and the left terminal-link delay was always shorter, but the left delay varied from FI 5 s to FI 40 s across four conditions. Pigeons' initial-link preference for the left (shorter, more immediate) key was a negatively accelerated, increasing function of absolute delay.

The initial link associated with the shorter terminal-link delay did not change in McEwen's (1972) experiment; the shorter delay was always on the left. However, each subject experienced pairs of delays in a different order, so the observed terminal-link effect could not have been due to amount of training alone. Williams and Fantino (1978) and Fantino and Royalty (1987) replicated the terminal-link effect of greater sensitivity to relative immediacy with longer absolute delays in concurrent-chains experiments in which the location of the shorter delay changed across experimental phases. Lower absolute rates of secondary reinforcement and lower absolute immediacies have opposite effects on preference: lower and higher sensitivity to immediacy ratio in concurrent chains, respectively.

1.2.5 Absolute reinforcer values and sensitivity in general

Absolute initial- and terminal-link duration affect sensitivity to relative immediacy in concurrent chains. In addition to those effects, the absolute values on other reinforcer dimensions also affect responding in experimental situations. To determine effects of absolute immediacy, magnitude and rate on sensitivity to relative immediacy, Logue and Chavarro (1987) trained pigeons in three consecutive concurrent-chains experiments in which absolute terminal-link delay, reinforcer duration and terminal-link entry rate were changed across conditions. Their Experiments 1 and 3 replicated initial and terminal link effects, respectively. In their Experiment 2, pigeons responded in concurrent chains with

VI 8-s initial links and FT 6-s terminal links. Reinforcer magnitude was changed via duration (in seconds) of access to grain per reinforcer. The larger magnitude was fixed at 3 times that of the smaller magnitude, which ranged from 0.75 to 5.25 s. Preference for the larger reinforcer was less extreme in conditions with larger absolute magnitude, indicating that sensitivity to relative magnitude decreases as absolute magnitude increases.

The effect of absolute magnitude on sensitivity to relative magnitude is comparable to the terminal-link effect. In both cases, sensitivity to the dimension under scrutiny is negatively related to absolute reinforcer access (Neuringer, 1967), duration of access to food per amount of time spent responding. No studies have shown any effects of absolute immediacy or magnitude on sensitivity to other reinforcer dimensions. Instead, Squires and Fantino (1971) and Navarick and Fantino (1976) found that manipulating absolute delay had no systematic effect on sensitivity to relative reinforcer rate or magnitude, respectively. Similarly, experimenters manipulating absolute magnitude in concurrent schedules and concurrent chains (respectively) reported no significant effects on sensitivity to relative rate (McDevitt & Williams, 2003) or immediacy (Grace, 1999; Ong & White, 2004).

Alsop and Elliffe (1988) trained pigeons on concurrent VI VI schedules. Across conditions, they varied the overall rate of reinforcement across conditions from 0.22 to 10 reinforcers per minute. For each of 6 overall rates of reinforcement, pigeons experienced the following reinforcement ratios: 8:1, 4:1, 1:1, 1:4 and 1:8. Alsop and Elliffe reported that sensitivity to reinforcement rate was greater with higher rates of reinforcement. Whereas higher absolute reinforcer access ratios (shorter delays and larger magnitudes)

produce lower sensitivity to relative immediacy and magnitude, sensitivity to relative reinforcement increases as the overall rate of reinforcers increases.

Unlike absolute terminal-link immediacy and magnitude, the absolute or overall rate of reinforcement has been shown to affect sensitivity to other reinforcer dimensions. The initial-link effect (Fantino, 1969) is an example of this. Davison (1988) trained pigeons on equal, concurrent VI VI schedules in which reinforcer magnitude was 10 s on one key and 3 s on the other. Davison reported that increasing the overall rate of reinforcement had the opposite effect on pigeons' sensitivity to relative magnitude as on sensitivity to relative rate (Alsop & Elliffe, 1988) or immediacy (Fantino, 1969): sensitivity of response and time allocation to relative magnitude decreased as programmed reinforcers per hour increased.

Effects of absolute rate, immediacy, magnitude and other dimensions on responding must be addressed in theories of choice. Logue and Chavarro (1987, p. 280) stated that the primary goal of their experiments was to "examine the assumption of the matching law that relative and not absolute values of reinforcement control preference." Davison (1988) described his results as an interaction between reinforcer rate and magnitude. Do their results imply that the matching law is wrong? If generalized matching is to be taken as a complete account of choice, the implication is that *only* relative reinforcer ratios determine preference and that absolute rate, immediacy and magnitude should have no effect on responding. However, Rachlin (1971) suggested that matching is a restatement of assumption rather than an empirical law; hence it is not subject to falsification by experimentation. Killeen (1972) explained that it is possible to consider the matching law as an empirical description of results, or as a means of quantifying the utility of a reinforcer.

In the latter case, the matching law serves a definitive function and is not subject to disproof. Investigations of choice between qualitatively different reinforcers (Hollard & Davison, 1971; Miller, 1976) exploit the matching law in this way. In the former case, the disproof of a particular generalized matching equation would not suggest that the principle of matching is incorrect. Instead, Killeen argued that it should be interpreted as an indication that the correct transformation of the reinforcer dimension under scrutiny was not identified by the particular equation that was applied.

That there are effects of absolute terminal-link entry rates, delays and values for other reinforcer dimensions demonstrates that other factors affect response allocation than those identified by the generalized matching relation alone. These effects do not invalidate matching. Equation 1.7 provides an excellent quantitative description of responding in many free-operant choice experiments. Stripped of any causal implications, it is not subject to verification or falsification; it can only be empirically refined.

1.3 Quantitative models of choice between delayed rewards

The concatenated generalized matching relation provides a descriptive but atheoretical account of choice between delayed rewards. Many quantitative models have arisen from studies examining the matching law. These quantitative models are grounded in testable assumptions about choice. The most successful of the steady-state models for choice between delayed rewards are delay-reduction theory (DRT; Fantino, 1969), the contextual choice model (CCM; Grace, 1994) and the hyperbolic value-added model (HVA; Mazur, 2001).

1.3.1 Delay-reduction theory

Generalized matching does not predict the initial-link effect. Fantino (1969) proposed that, rather than being directly proportional to reinforcement, initial-link response allocation in concurrent chains is a function of the reduction in delay to reward signaled by the stimulus change that occurs upon terminal-link entry:

$$\log\left(\frac{B_L}{B_R}\right) = \log\left(\frac{T - t_L}{T - t_R}\right).$$

Equation 1.8

In Equation 1.8, T is the mean total time that elapses between initial link onset and reinforcer delivery; t_L and t_R represent time from terminal-link onset to reinforcer delivery for left and right alternatives, respectively. The value of T is the sum of left initial- and terminal-link durations multiplied by the proportion of terminal links arranged on the left, plus the sum of right initial- and terminal-link durations multiplied by the proportion of terminal links arranged on the right. Fantino predicted that Equation 1.8 would describe response allocation in concurrent chains as long as t_L and t_R were both less than T. Equation 1.8 correctly predicts effects of absolute initial- and terminal-link duration.

Squires and Fantino (1971) noted that when terminal-link delays are equal, Equation 1.8 predicts indifference regardless of initial-link duration or terminal-link entry rates. Indifference is a counterintuitive prediction in concurrent chains with unequal initial-link schedules but equal terminal-link schedules. They hypothesized that response allocation is determined by a multiplicative concatenation of terminal-link entry rate and delay reduction. In logarithmic terms:

$$\log\left(\frac{B_L}{B_R}\right) = \log\left(\frac{R_L}{R_R}\right) + \log\left(\frac{T - t_L}{T - t_R}\right).$$

Equation 1.9

The Squires and Fantino formulation of delay reduction theory (DRT) is similar to concatenated matching in that it assumes different reinforcer dimensions combine multiplicatively. What is different about Equation 1.9 is the way that conditioned reinforcement value is construed as signaled reduction in delay to reinforcement relative to an overall mean time between the onset of initial links and food delivery (rather than reinforcement). As a theory of conditioned reinforcement, DRT can explain many of the behaviors associated with choice between delayed rewards, including some that are not described by matching alone, such as the initial- and terminal-link effects. Fantino, Preston and Dunn (1993) reviewed over 20 years of research evaluating DRT and determined that it can explain a variety of choice phenomena.

1.3.2 The contextual-choice model for choice

According to Grace's (1994) contextual choice model (CCM), conditioned reinforcement value is a function of relative terminal-link immediacy (as in the generalized matching case of Equation 1.5). However, Grace hypothesized that, in addition to potential nonidentity sensitivities to individual conditioned reinforcer dimensions, sensitivity of response allocation to conditioned reinforcement value as a whole is affected by relative time spent in initial and terminal links:

$$\log\left(\frac{B_L}{B_R}\right) = \log b + a_R \log\left(\frac{R_L}{R_R}\right) + \left(\frac{Tt}{Ti}\right)^k \left[a_D \log\left(\frac{1/D_L}{1/D_R}\right) + a_X \log\left(\frac{X_L}{X_R}\right)\right].$$

Equation 1.10

The log-transformation of Grace's original formulation of CCM, Equation 1.10, represents a theoretical modification to concatenated generalized matching. According to CCM, all conditioned reinforcer ratios (i.e., everything other than terminal-link entry rate) are raised to an additional exponent, $(Tt/Ti)^k$. *Tt* is average terminal-link duration, *Ti* is average initial-link duration and *k* is a scaling parameter. The major unique theoretical assumption of CCM is that the degree to which conditioned reinforcement value controls choice is determined by relative time spent "in the presence of terminal-link discriminative stimuli…[as in delayed-discrimination learning] the effectiveness of immediate conditioned reinforcement increases as the absolute delay to primary reinforcement increases." (p. 119). CCM correctly described a wide range of behavioral phenomena observed in concurrent chains, including initial- and terminal-link effects.

1.3.3 The hyperbolic value-added model

Mazur's (2001) hyperbolic value-added model (HVA) states that choice between delayed rewards is determined by the relative change in value signaled by terminal-link onset. According to DRT, relative reduction in delay signaled by terminal-link onset is the salient determiner of conditioned reinforcement value. By contrast, in HVA, response allocation is determined by the relative amount of value added by terminal-link onset. According to HVA, the values of initial links and of each terminal link are determined by the hyperbolic-decay equation Mazur (e.g., 1984, 1986, 1991) had applied to results from many experiments on discrete-trial choice successfully:

$$V = \sum_{i=1}^{n} P_i \left(\frac{M}{1 + KD_i} \right).$$

Equation 1.11

In Equation 1.11, the value, V, of an alternative signalling a VI schedule is determined by the sum of all possible delays with each delay, D_i, weighted according to the probability of that delay occurring in the schedule, P_i. The value of each delay is determined by its reinforcer magnitude divided by $1 + KD_i$, where K is a discounting parameter that determines how much value decreases as delay increases. This hyperbolic-decay equation states that an average of the values associated with all possible delays, weighted according to the probability of that delay occurring in the schedule, determines the total value of a VI (and, by extension, FI, VT, FT or percentage reinforcement) schedule.

Like DRT, CCM and other theories of choice inspired by matching (Davison & Temple 1973; Killeen 1982), HVA assumes that relative responding is a function of relative value. In concurrent-chains schedules, Mazur (2001) assumed that delay to primary reinforcement determined value in initial as well as terminal links, and that rather than relative terminal-link value, response allocation was a function of the relative value added by the onset of a terminal link:

$$\log\left(\frac{B_L}{B_R}\right) = \log b + a_R \log\left(\frac{R_L}{R_R}\right) + \log\left(\frac{V_{tL} - a_D V_i}{V_{tR} - a_D V_i}\right).$$

Equation 1.12

Equation 1.12 is a log transformation of Mazur's (2001) Equation 6. Variables V_i, V_{tL} and V_{tR} denote initial link, left terminal link and right terminal link value, respectively. Terminal link values are obtained from a version of Equation 1.11 in which each D_i is the delay from terminal-link onset to primary reinforcement. The value of initial links, V_i, is calculated using a version of Equation 1.11 in which each D_i is the delay from initial-link onset to primary reinforcement.

1.3.4 Comparing quantitative models of steady-state choice

Contemporary theories of steady-state concurrent chains choice, DRT, CCM and HVA all provide accounts of choice between delayed rewards that are informed by the matching law. These 'matching' models of steady-state choice share common assumptions about the relationship between time to food and response allocation with generalized matching, namely that degree of relative value directly determines strength of preference; i.e, response allocation matches relative value. All of them can be generalized to include parameters to account for differences in bias and sensitivity to individual reinforcer dimensions. To account for the initial- and terminal-link effects, formulations of DRT, CCM and HVA (but not of the generalized matching law) all include some kind of scaling parameter applied to conditioned reinforcement value globally. There are also differences between the three models – a calculus of value operates differently for each one.

Generalized matching is tautologous (Rachlin, 1971; Killeen, 1972); extremely useful descriptively but lacking explanatory power. By contrast, DRT, CCM and HVA are theoretical as well as descriptive because each makes additional assumptions about what determines conditioned reinforcement value. Conditioned reinforcement value is a function of the reduction in expected delay to food signaled by the stimulus change (terminal-link onset) in DRT, and similarly of the value added by that reduction in expected delay in HVA. Like generalized matching, CCM assumes that log ratios of conditioned reinforcer dimensions combine additively and independently to determine log response ratios. Unlike generalized matching, CCM additionally scales those log ratios according to $(Tt/Ti)^k$, the proportion of time spent in terminal relative to initial links. Because closed-form equations exist for all three models and distinguishing quantitative features are identifiable for each, formal comparisons of DRT, CCM and HVA ars possible and worthwhile.

A number of criteria can be used to evaluate different quantitative models. Parsimony is desirable – if two models provide the same goodness of fit, the simpler model (the one with fewer parameters) is better. If all models have the same number of parameters, they can be quantitatively compared directly using a measure of goodness of fit. Comparing models with different numbers of parameters is more complicated. If it is possible to do so without compromising theoretical integrity, restricted models can be parameterized or full models can be simplified so that the quantitative formulations of the different models have the same number of parameters. Alternatively, relative goodness of fit statistics can be used. For example, *F* ratios compare VAC of nested models (in which the full model includes all of the parameters used in the restricted model). Ideally, a good quantitative model should generalize. The range of values over which its predictions are valid (or calculable) should include range for which there are empirical results. Adaptation or simplification of the model in a manner consistent with its theoretical assumptions should generate predictions for other procedures. A good quantitative model not only describes existing data, but it makes testable predictions about novel situations. However, the benefits of generality must be weighed against a model's predictive validity, as there is often a tradeoff between the two.

Davison (1987) evaluated three quantitative models describing choice between delayed rewards by estimating parameters and VAC when he applied each model to group mean data from ten archival data sets. Davison estimated slope and intercept parameters for the Squires and Fantino (1971) formulation of DRT, Killeen's (1982) Incentive theory and Davison and Temple's (1973) alternative model. Davison (1987) did not include a bias parameter in any model and fixed an additional parameter of Incentive theory at a previously determined (Killeen, 1982) appropriate value. With these restrictions, all three

models had the same number of parameters, therefore it was appropriate to compare them quantitatively using VAC alone. One model – DRT – fitted data from experiments with VI terminal-link schedules better than fixed terminal-link data, but the difference was not significant. Both other models fitted fixed terminal-link data significantly better than variable terminal-link data. Across the ten data sets, applications of DRT, Incentive theory and the alternative model produced average VACs of 0.68, 0.64 and 0.69, respectively. Davison concluded "60-70% of the data variance in any set of data succumbs to almost any rational model… that makes correct ordinal predictions," and therefore that "Overall, all three [models] described initial-link response allocation poorly." (p. 234). He advocated that rigorous application of models and efforts to establish parameter invariance should be *a priori* requirements of acceptance of quantitative models of choice.

Grace (1994) applied CCM (Equation 1.10) to 92 data sets from 19 published two-alternative concurrent chains studies in which terminal links were either both fixed or both variable. Included studies presented pigeons with concurrent chains in which relative and absolute initial- and terminal-link values changed. Across the 19 studies, it accounted for an average of 91% of the variance in those data sets. Grace also applied equations describing Squires and Fantino (1971) formulation of DRT, Killeen's (1982) Incentive Theory, and Davison and Temple's (1973) alternative model to the same data. Those models accounted for an average of 54, 51 and 55% of variance, respectively. A quantitative comparison of the models based on goodness of fit would be inappropriate because CCM included several additional parameters. However, the large difference in VAC for CCM compared to VAC for the specific formulations of DRT, Incentive theory and Davison and Temple's (1973) alternative model provides qualitative support in favor of CCM as a superior account of choice between delayed rewards.

CCM provided a superior account of the 92 data sets Grace (1994) reanalyzed compared to the Squires and Fantino (1971) formulation of DRT, but CCM had four more free parameters than DRT. In addition to outlining HVA, Mazur (2001) presented a generalized formulation of DRT with additional parameters for sensitivity to terminal-link entry rate, immediacy and conditioned reinforcement value, and for bias. CCM and Mazur's (2001) generalized formulation of DRT have the same number of parameters; therefore, it is appropriate to directly compare goodness-of-fit for the three models. Mazur reported parameter estimates and goodness of fit (proportion of variance accounted for, VAC) from CCM, HVA and DRT fitted to the 92 data sets in Grace's (1994) sample. Goodness of fit estimates for CCM, HVA and DRT were 0.91, 0.90 and 0.83, respectively. All three models made fairly accurate predictions of performance in concurrent chains.

Mazur (2004) compared predictions of parameterized formulations of DRT, CCM and HVA for performance in concurrent-chains schedules in which initial- or terminal-link durations were varied, but immediacy ratio did not change. Mazur noted that each model predicts a different functional relationship between initial-link response allocation and initial- or terminal-link duration. In Mazur's Experiment 1, four pigeons experienced concurrent chains reinforced according to VI initial and terminal links. Terminal-link entry was dependently scheduled according to a single VI 20-s schedule. The shorter terminal link was reinforced with access to food according a VI schedule ranging from 0-7 s across conditions. The longer terminal-link delay was always 3 times the shorter terminal-link delay. Response proportion was a "curvilinear" function of shorter terminal-link delay, a result consistent with the predictions of DRT and HVA but not CCM.

In Mazur's (2004) Experiment 2, the same subjects experienced concurrent chains with VI initial and terminal links, in which terminal-link schedules were VI 2 s and VI 12 s. The duration of the single VI schedule that operated in initial links varied from 10 to 450 s across conditions. All three models predicted that initial-link response proportion for the shorter terminal link would be a negatively accelerated monotonically decreasing function of initial-link duration. However, whereas DRT and CCM predicted an asymptote at indifference, the asymptotic level of preference predicted by HVA was greater than indifference. Results were consistent with the prediction of HVA that preference would not reach indifference.

Although DRT, CCM and HVA all account for performance in concurrent chains with VI initial links and delayed (VI, FI, VT or FT) terminal links, they differ in how and under what circumstances they have been applied to other procedures. DRT was developed to explain choice between delayed rewards, and makes no predictions about choice in concurrent schedules. However, Abarca and Fantino (1982; Fantino & Abarca, 1985) developed a model based on DRT that predicts choice in laboratory and field studies of foraging. When *Tt* is 0, CCM reduces to generalized matching. For this reason, CCM can be considered a theory that describes choice between two concurrent schedules as well as between two concurrent chains. The hyperbolic equation used to quantify value in HVA was first applied to results of delay (Mazur, 1986; Mazur, Stellar & Waraczynski, 1987; Myerson & Green, 1985; Rodriguez & Logue, 1988) and probability (Mazur, 1989, 1995; Mazur & Romano, 1992) discounting. Thus, in a sense HVA can be considered an extension of discounting theories. All three quantitative models can be adapted to account for data from other paradigms.

DRT, CCM and HVA are quantitative models of choice between delayed rewards that describe initial-link response allocation under a variety of concurrent-chains conditions. They all can be applied to other experimental paradigms. However, they are all steady-state models, developed to describe behavior that has stabilized. None of them address the question of how choice develops or adapts as situations change.

1.4 Choice in transition

A quantitative model of choice that is capable of describing how response allocation adapts when contingencies change is desirable for several reasons. One is ecological validity – choice outside the operant chamber occurs in necessarily dynamic environments. Relying on frequent changes in reinforcer contingencies can be a pragmatic way of exposing subjects to many different reinforcer ratios, which can provide more insight about the "correct transformation" (Killeen, 1972) relating response allocation to that reinforcer dimension. Importantly, in order to identify and understand the process or processes that generate choices, observing the course of change in preference to stability is critical. Steady-state theories of choice provide accounts of the end result of the process but are silent regarding the process itself. Any plausible explanation of choice between delayed rewards must account for response allocation dynamics if it is to shed any light on the generative process.

1.4.1 Molecular models of concurrent-schedules choice

Developers of early dynamic models of choice considered how generalized matching might be extended to explain concurrent-schedules choice in transition. For example, two quantitative models, melioration (Herrnstein, 1982; Vaughan, 1981, 1982, 1985), and a kinetic model (Myerson & Hale, 1988; Myerson & Miezen, 1980) predict

matching at equilibrium, but arrive at matching via different acquisition trajectories. The premise of melioration theory is that in concurrent schedules, subjects allocate responses to the alternative offering the richer local rate of reinforcement. Melioration theory predicts that adjustments in preference should follow a linear trajectory. The melioration process equalizes local reinforcement rates. By contrast, Myerson and Miezen's (1980) kinetic model equilibrates probability of switching from responding on one alternative to the other (Myerson & Hale, 1988). The kinetic model predicts negative exponential and logistic preference adjustment trajectories for VI and VR concurrent schedules, respectively. Both melioration and the kinetic model represent interesting attempts to describe the process that generates matching, however, neither has been extended to choice between delayed rewards.

1.4.2 A local effects model: Every reinforcer counts

Recently, Davison and Baum (2000; also Baum & Davison, 2004, 2009; Davison & Baum, 2002, 2003, 2006, 2007) presented a novel approach to free-operant choice analysis. It involves quantifying preference and sensitivity as functions of successive reinforcers.

Davison and Baum (2000) trained six pigeons in a two-key concurrent schedule with seven components. In all components, VI schedules operated independently on each key. Reinforcer ratios in the seven components varied between 1:27 and 27:1. Davison and Baum programmed all components to occur each session in pseudorandom order. Components were separated by blackouts but not differentially signaled. Preference adjusted to the unsignaled reinforcer ratios rapidly and stabilized within components, albeit at lower levels than have been reported for the same ratios in steady-state experiments

(Baum, 1979; Taylor & Davison, 1983). Davison and Baum plotted log response ratio as a function of successive reinforcers in a component according to the sequence of their left:right occurrence. They reported that the magnitude of the effect of reinforcement on sensitivity to reinforcer ratio diminished with each subsequent reinforcer to the preferred key, but that a reinforcer delivered on the non-preferred key had a large effect at any point in a sequence. Baum and Davison (2009) showed that a linear operator model could describe local effects of reinforcement and nonreinforcement on response allocation.

Davison and Baum (2002) subsequently applied this procedure and analytic approach to investigate effects of blackout duration and extinction components. They reported that a degree of residual preference from the previous component persisted 60s into blackout or extinction, indicating longer-term effects of reinforceer ratios than hypothesized earlier by Davison and Baum (2000). Baum and Davison (2004) observed within-component patterns of visit duration that have been called fix and sample in steady-state research (Baum, Schwendimann, Bell, 1999), with pigeons fixing on whichever alternative was last reinforced.

Using similar procedures, Davison and Baum have investigated effects of relative magnitude (Davison & Baum, 2003), and conditional (Davison & Baum, 2006) and delayed (Davison & Baum, 2007) reinforcement. Effects of relative immediacy and magnitude were similar to, but smaller than, effects of relative rate of reinforcement. When Davison and Baum (2006) paired food presentation with a conditioned stimulus (the hopper light), occasional presentations of the hopper light alone had similar, though smaller, effects on performance as did food. When the hopper light was never presented concurrently with food, its effects on performance opposed those of food. Davison and Baum (2006, 2007)

concluded that the effects of food on responding in concurrent schedules did not constitute strengthening of an association, but rather that they could be better-described as selection by consequences. Although these analyses have not been applied to concurrent-chains performance, the findings regarding delayed and conditioned reinforcers suggest that an extension of Baum and Davison's (2009) linear-operator local-effects model could be applied to initial-link responding.

1.4.3 Empirical approaches to studying choice in transition

Many researchers have investigated choice in transition using different empirical approaches. They have produced a body of evidence that rate of preference acquisition can be affected by many of the same factors as preference in steady-state choice procedures. For example, Bailey and Mazur (1990) found that in a discrete-trials choice procedure with probabilistic reinforcement, rate of preference acquisition depended on the ratio of probabilities; more extreme ratios produced more extreme preference. Contrary to the predictions of the kinetic model (Myerson & Hale, 1988), Mazur and Ratti (1991) obtained the same result in a free-operant choice procedure.

Changes in reinforcement can have different effects within and across sessions. Mazur (1995, 1996) reported 'spontaneous recovery' of previous levels of preference in dependent concurrent VI-VI schedules. Across blocks of sessions, the proportion of reinforcers assigned to an alternative (Mazur, 1995) or the probability of reinforcement (Mazur, 1996) was changed. Pigeons' preference adjusted to reflect post-transition contingencies. However, at the beginning of the first few post-transition sessions, preference reverted to pre-transition levels. Because this spontaneous recovery of pre-

transition preference did not persist, Mazur was able to model response allocation at the beginning of sessions using a weighted average of contingencies from previous sessions.

Compared to the relative abundance of steady-state research on concurrent-chains choice, few experiments investigate how initial-link response allocation adapts to changes in reinforcer contingencies. Mazur, Blake and McManus (2001) investigated effects of sudden changes in terminal-link entry rate or delay on rate of preference acquisition in concurrent-chains schedules. They reported gradual adjustments of initial-link response allocation after the percentage of reinforcers allocated to each initial link or terminal-link delays were switched. Rate of acquisition and stable level of preference were both more sensitive to changes in terminal-link entry rate than to changes in terminal-link delays. This result was unexpected because changes to delays should be more easily discriminated since they are detectable after a single trial, whereas the completion of multiple trials would be necessary to detect a change in entry rate. Because response allocation typically did not stabilize for several post-transition sessions, Mazur, Blake and McManus speculated that degree of discriminability was not the rate-limiting factor in choice acquisition.

Mazur (2002) trained pigeons in concurrent chains with dependent VI initial links. Both terminal links were either FT 1 s or FT 20 s. Terminal-link entry rate changed across conditions, which lasted 5-9 sessions. Terminal-link duration did not affect rate of acquisition. While changes in preference were more extreme with more dramatic changes in terminal-link entry rate, initial-link time to response allocation stability was similar for all pairs of pre- and post-transition entry rates. Mazur concluded that although the lack of an effect of delay on sensitivity to relative rate is contrary to predictions of conditioned

reinforcement models for choice (such as DRT), there are several possible explanations that could reconcile these results with principles of conditioned reinforcement.

To compare linear-operator and memory-representational approaches to quantitatively modeling choice, Grace (2002a) employed a similar successive-reversals design. Although the linear-operator and memory-representational models were roughly consistent with matching-based and timing-based theories of choice, the quantitative models used were atheoretical in nature. Pigeons responded in concurrent chains with dependent VI initial links. Terminal links could be both fixed, both variable or one fixed and one variable, with immediacy ratios of 1:2, 2:1, 1:4 and 4:1. Terminal-link schedules remained in effect for 20 sessions. The initial link associated with greater terminal-link immediacy alternated with each successive reversal, and terminal-link schedule values and schedule types (fixed or variable interval) could change as well. Consistent with predictions of both models, Grace (2002a) reported that rate of preference acquisition was faster when pre-transition terminal links were fixed and when post-transition terminal links were variable. However, whereas the memory-representational model predicted no effect of absolute terminal-link duration on rate of acquisition, the linear-operator model predicted that, for the same immediacy ratio, acquisition should be faster with longer absolute terminal-link delays. Results were consistent with the linear-operator prediction.

Berg and Grace (2004) used a successive-reversals design to investigate effects of absolute initial-link duration on choice in transition. Berg and Grace presented pigeons with concurrent chains with VI initial links and FI terminal links. Terminal-link delays were always FI 8 and 16 s, and the location of the shorter delay switched every 20 sessions. Initial-link schedules were either short (VI 8 s) or long (VI 24 s). They reported that

acquisition of preference was faster when pre-transition initial links were short and when post-transition initial links were long.

Many of the features that are associated with greater sensitivity in steady-state procedures are often associated with more rapid acquisition in experiments in which contingencies change across blocks of relatively few (5-20) sessions. Such features include relative probability of reinforcement (Bailey & Mazur, 1990) and absolute and terminal-link duration (Grace, 2002a). That Mazur (2002) reported no effect of terminal-link duration on rate of acquisition when terminal-link entry rates changed is also consistent with steady-state sensitivity results. It should be possible to exploit this information to design a procedure in which acquisition can occur very rapidly.

1.4.4 Rapid acquisition experiments

To study choice in transition, Davison and colleagues (Hunter & Davison, 1985; Schofield & Davison, 1997) and Grace and colleagues (Christensen & Grace, 2008, 2009a; Grace, Bragason, McLean, 2003; Grace & McLean, 2006, Maguire, Hughes, & Pitts, 2007; Ta, Pitts, Hughes, McLean & Grace, 2008) have trained pigeons in concurrent-schedule or concurrent-chain procedures in which reinforcer contingencies changed across sessions. Grace and colleagues use short initial links and FI terminal links, known to be conducive to overmatching in steady-state concurrent chains. The typical result of concurrent-schedule and concurrent-chain experiments is that performance adapts to the daily changes in contingencies and stabilizes within the session. In other words, preference is rapidly acquired. Grace and colleagues refer to experiments in which contingencies are changed across sessions as *rapid acquisition* experiments.

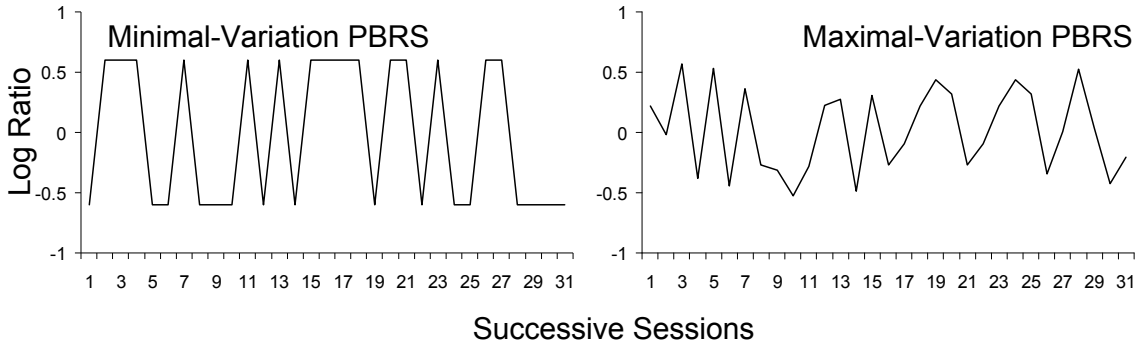

Figure 1.3. Sample minimal- and maximal-variation pseudorandom binary sequences.

To characterize dynamic properties of concurrent-schedules performance quantitatively, Hunter and Davison (1985) trained pigeons on two-key independent concurrent VI VI schedules in which reinforcement ratio changed across sessions according to a 31-step pseudorandom binary sequence (PRBS). The left panel of Figure 1.3 shows one "minimal-variation" PRBS of the two used in the experiment; programmed reinforcement ratios were always one of two reciprocal values, 1:4 or 4:1. Reinforcement was scheduled independently for each key, so obtained reinforcement ratios differed from programmed ratios, however, there were no sequential dependencies. Hunter and Davison (1985) reported that log response ratios calculated from each session were determined by log reinforcement ratios in effect in the same session (i.e., Lag 0 ratios). However, sensitivity was lower than typically reported in steady-state concurrent-schedules experiments.

Schofield and Davison (1997) trained pigeons on concurrent VI VI schedules in which scheduled reinforcer ratios were either of two reciprocal values (1:2 and 2:1, 1:4 and 4:1, or 1:8 and 8:1 across conditions). Across sessions, a 31-step PRBS determined whether the left or the right key would be reinforced more frequently. To assess degree of control exerted by reinforcer ratios from current and previous sessions on choice, Schofield and Davison applied a generalized matching model that used obtained log reinforcer ratios

from the current (Lag 0) session and from the nine sessions before the one in which the measurements were taken (Lags 1 through 9) to log response and time allocation ratios:

$$\log\left(\frac{B_{Ln}}{B_{Rn}}\right) = a_0 \log\left(\frac{R_{Ln}}{R_{Rn}}\right) + a_1 \log\left(\frac{R_{L(n-1)}}{R_{R(n-1)}}\right) + \ldots + a_9 \log\left(\frac{R_{L(n-9)}}{R_{R(n-9)}}\right) + \log b$$

Equation 1.13

Subscript *n* refers to the current session. Schofield and Davison found that after pigeons had been exposed to three presentations of the PRBS (i.e., after 93 sessions), sensitivity was greatest for Lag 0 ratios and close to zero for all others, suggesting that performance was determined by reinforcer ratios in the current session. Schofield and Davison (p. 219) concluded that using the PRBS was an appropriate method of obtaining "fast discrimination of sensitivity to reinforcement."

Grace, Bragason and McLean (2003) used the same procedural and analytic approach to studying choice between delayed rewards in transition, and they obtained similar results in concurrent chains. In their Experiment 1, the left terminal-link delay was always FI 8 s and the right terminal link was either FI 4 s or FI 16 s, determined by the minimal-variation 31-step PRBS depicted in the left panel of Figure 1.3, except that log immediacy ratios were 2:1 and 1:2. Multiple regression analyses showed that after three presentations of the PRBS, Lag 0 sensitivity to immediacy coefficients were positive and significant for all subjects and higher-lag coefficients were near zero.

In Grace, Bragason and McLean's (2003) Experiment 2, left terminal-link delay was always FI 8 s. Whether the right terminal-link delay was longer or shorter than the left was determined by the same 31-step PRBS, and schedule values were derived from immediacy ratios generated pseudorandomly from a potentially-infinite population of

values. The right panel of Figure 1.3 shows a typical sequence of immediacy ratios from Experiment 2. Grace, Bragason and McLean that Lag 0 log immediacy ratios determined log response ratios calculated over all 72 initial links of each session. There was no systematic effect of higher-Lag immediacy ratios on response allocation. Grace, Bragason and McLean reasoned matching-based theories of choice such as CCM predicted that the functional relationship between log immediacy ratio and log response ratios should be continuous. However, inspection of scatterplots comparing response ratios to right-key terminal-link delay (equivalent to log immediacy ratio in this Experiment) suggested a tendency for data points to fall into two separate clusters. Grace, Bragason and McLean noted that the observed clustering was consistent with categorical discrimination.

Grace and McLean (2006) replicated Grace, Bragason and McLean's (2003) results, including the clustering of response allocation. Grace and McLean's "minimal variation" condition was identical to Grace, Bragason and McLean's Experiment 1. Their "maximal variation" condition was identical to Grace, Bragason and McLean's Experiment 2, except that both terminal-link delays changed across sessions. Grace and McLean reported that there was no systematic difference in sensitivity to Lag 0 log immediacy ratios between minimal- and maximal-variation conditions. They proposed a 'decision model' (Section 1.8) that assumed that a categorization of each experienced delay as long or short decreased or increased response strength, which determined response allocation.

1.5 Delayed rewards and the construct 'expectancy'

Contemporary steady-state models of choice, including but not limited to conditioned reinforcement models such as DRT, CCM and HVA, trace their heritage to Herrnstein's (1961) initial work on the matching law. However, the concept of 'delay' is as

much the provenance of temporal control as of matching research. Historically predominant theories of timing were developed to explain patterns of responding in simple interval schedules; some of them have been extended to predict choice behavior as well. These theories rely on the construct 'expectancy.' In theories of timing, expectancy refers to the degree of anticipation of reinforcement or the (mathematically) expected remaining delay to reinforcement. Theories typically assume that previously experienced intervals between time markers and reinforcement, or subjective perceptions thereof, determine expectancy. Extensions of theoretical accounts of timing behavior include explanations of choice in transition. By capitalizing on an assumed intrinsic relationship between preference and temporal control, expectancy-based models may provide more complete quantitative accounts of behavior than models that were developed to explain choice alone.

1.5.1 Measuring temporal control

Skinner (1938) identified time as a "discriminable continuum" and distinguished temporal discrimination from sensory discrimination that is not strictly temporal but has temporal properties because behavior necessarily occurs in time. According to Skinner (1938, p. 265), temporal discrimination occurs when "the temporal correlation [between the presence of a stimulus and delivery of a reinforcer] makes it possible to single out a given point on the continuum established by the prolonged presence of a stimulus." The peak procedure (Catania, 1970; Roberts, 1981) is a means of measuring temporal control in interval schedules.

1.5.2 Empirical research on interval timing

The extent to which a subject's behavior is determined by the schedule value is an estimate of temporal control. In memoryless VI schedules, the arranged conditional

probability of reinforcement is constant – rate of reinforcement is the salient independent variable. Therefore, response rate should not change as a function of time since reinforcement or time since trial onset. In FI schedules, the salient independent variable is time to reinforcer availability, which does not change across trials. Thus, the pattern of responding as a function of time elapsed in a trial can come under the control of the conditional probability of reinforcement in both FI and VI schedules.

In a "classic" paper, Catania and Reynolds (1968) quantitatively analyzed responding in interval schedules of reinforcement in six experiments. In Experiment 1, they compared overall rates of responding (the total responses/total time) as a function of scheduled rate of reinforcement. Pigeons were reinforced according to VI schedules drawn from arithmetic progressions with mean intervals ranging from 12 to 427 s. They reported a monotonically increasing, negatively accelerated relationship between response rate and schedule value. They also evaluated responding as a function of conditional probability of reinforcement. They found that local response rate increased as a function of time since food, and the rate of increase was schedule-dependent, with shortest intervals producing the most dramatic increases. They concluded that "local" response rates were sensitive to the conditional probability of reinforcement. They noted that normalized temporal patterns of responding were timescale invariant in that they superposed. Taken together, these results provide evidence of temporal control over responding in VI schedules.

In Experiments 4-6, Catania and Reynolds (1968) examined properties of responding in fixed-interval schedules. In Experiment 4, they trained four pigeons on single FI 50 s, FI 200 s and FI 30 s schedules, across conditions, in that order. There was no intertrial interval (ITI); the timing of each trial began from the end of the previous

reinforcement. Catania and Reynolds reported that, whereas average response rate did not change systematically with FI duration (in contrast to results with arithmetic VI schedules), "terminal" response rates calculated over the end of each trial were positively correlated with reinforcements per hour. For all subjects and schedule values, response rates increased as a function of time to food, and when relative response rates were plotted as a function of relative time to reinforcement, the resultant temporal gradients superposed.

Dews (1970) and others have found that in FI schedules, the "global" response rate is proportional to time to food. Dews described performance under FI schedules of reinforcement. In both VI and FI schedules, pigeons' local response rates are sensitive to conditional probability of reinforcement. The global (or total) rate of responding is sensitive to global rate of reinforcement. That both "local" and "global" response rates are sensitive to time to food suggests that responding in FI schedules is under temporal control.

Schneider (1969) investigated local patterns of responding on individual trials of FI procedures. Six pigeons responded in FI schedules in which schedule value changed across conditions and ranged from 16 to 512 s. To assess patterns of responding in individual trials, Schneider calculated interresponse times (IRTs). Although average local response rate increased gradually as a function of time since trial onset when averaged across many trials, response rate in single interfood intervals did not increase gradually. Instead, Schneider (1969, p. 679) characterized responding in the majority of intervals as a "two-state process, [beginning with] an extended pause after reinforcement, followed by a rapid acceleration to a high and approximately constant rate of response." Schneider noted that the first response following the post-reinforcement pause was not necessarily the point of maximum acceleration in rate, or breakpoint. To a first approximation, breakpoints in FI

"break-run" patterns of responding were normally distributed with a mean at about two-thirds of the FI duration. The characteristic sigmoidal temporal gradient observed in other FI experiments (for example, Catania & Reynolds, 1968) must be a result of averaging across trials.

1.5.3 Peak procedure

Catania (1970) examined the discriminative properties of interval duration. He described a procedure in which subjects' responses were reinforced with access to food after 10 s had elapsed since trial onset (marked by the reinstatement of a keylight stimulus following a 60-s intertrial blackout interval) on some trials. On other trials, reinforcement was omitted, the keylight remained on for a total of 48 s and responses had no effect. The probability that reinforcement became available after the FI duration elapsed was either 90% or 10%. In both cases, local response rate (based on data aggregated across trials) increased until scheduled time to reinforcement on reinforced trials and thereafter decreased if reinforcement did not occur.

Roberts (1981) reported five experiments in which he employed a modification of Catania's (1970) procedure to measure temporal discrimination. What is now referred to as the peak-interval (PI) procedure consists of two types of trials. In food trials, responding is reinforced according to FI schedules. No-food trials (empty trials in Roberts, 1981) end without reinforcement after a longer duration (usually many times longer than the FI duration) has elapsed. Responses on no-food trials have no effect.

Figure 1.4. Peak Procedure.

Figure 1.4 is a diagram of a peak procedure used with pigeons to study temporal control over responding. As in Figure 1.2, the horizontal axis represents time, circles represent keylights in an operant chamber, open triangles indicate reinforcement is available pending a keypeck response, and letters denote stimulus changes. Line segment AB represents a food trial, in which a single key is lighted except during reinforcer delivery and reinforcement is available according to an FI schedule. All keys are darkened during the ITI (line segment BC), which can be of fixed, variable or no duration. A pseudorandomly selected proportion of trials are no-food trials that end without reinforcement after a much longer duration.

In each of five experiments in which rats' leverpressing was reinforced with food according to a single PI or two mixed PI schedules, Roberts (1981) generated a response distribution (or frequency histogram) by aggregating responses over peak trials according to the time since trial onset when each response occurred for every subject and session. Response distributions were approximately normal with means at or near the scheduled time of reinforcer availability on food trials. Roberts calculated peak time as a measure of temporal control. Peak time is an estimate of the average time that has elapsed in no-food trials when the median response occurs. It can be calculated in different ways. Roberts took the time of the median response calculated over an interval from trial onset to double

the schedule value to be the peak time in his Experiment 1, and used an iteration technique to calculate peak time in subsequent experiments. Peak rate is the response rate at peak time. Roberts interpolated peak rate based on number of responses in the two histogram bins nearest to peak time. He was able to manipulate peak rate and peak time independently. PI schedule value, ITI duration and the omission of reinforcement on some food trials affected peak time but not peak rate. Proportion of 'food' trials and prefeeding affected peak rate but not peak time. The observed independence of peak time and peak rate is consistent with Schneider's (1969) characterization of FI responding as a two-state process in which responding occurs at a low rate during the break phase and a high, approximately constant rate during the run phase.

Gibbon and Church (1990, 1992) examined pigeons' response distributions from individual no-food trials of a PI procedure in which scheduled time to food on reinforced trials was either 30-s or 50-s. Response distributions were approximately normal when responding was aggregated across trials. Moreover, when relative response rates were plotted as a function of relative time (time elapsed/schedule value), distributions from PI 30 s and PI 50 s conditions superposed. Gibbon and Church (1990, 1992; also Church, Meck & Gibbon, 1994) noted that superposition of relative response distributions implies scalar variability in temporal patterns of responding: the standard deviations of distributions are a constant proportion of their means. Responding on individual no-food trials followed a break-run-break pattern corresponding to the break-run pattern characterized by Schneider (1969); it began at a low rate, switched abruptly to a high rate and, at a point after the schedule value had elapsed, switched just as abruptly back to a low rate.

Cheng and Westwood (1993) replicated Roberts' (1981) and Gibbon and Church's (1990, 1992) general findings. They evaluated pigeons' response distributions from individual no-food trials of a PI procedure in which the first peck after 12.5 s had elapsed was reinforced with access to food on 80% of trials and no-food trials of 25, 56.25 or 112.5 s comprised the remainder of trials. Responding on individual no-food trials followed a break-run-break pattern. Cheng and Westwood (1993) called the point at which responding switched from a low to a high rate the "start" time and the point when it switches from a high rate back to a low rate the "end" time, henceforward the "stop" time. To calculate start and stop times, Cheng and Westwood sorted responses from individual trials into a response distribution with 1.25 s bins. The start time was the lower boundary of the first of two consecutive filled bins, that is, bins containing responses. The stop time was the earlier boundary of the first of two consecutive empty bins occurring after the start time. Cheng and Westwood reported a positive correlation between start and stop times. Cheng, Westwood and Crystal (1993) extended Cheng and Westwood's (1993) work by presenting pigeons with different schedule values in various PI procedures. They calculated start and stop times in the same manner as Cheng and Westwood, and they also reported positive correlations between start and stop times and scheduled time to food.

Church, Meck and Gibbon (1994) calculated start and stop breakpoints in no-food trials using a different method than Cheng and Westwood (1993). Their goal was to identify the points in each trial at which maximum acceleration and deceleration in response rate occurred. Church, Meck and Gibbon (1994) trained 30 rats in a PI procedure. Each trial was independently determined to be a food or no-food trial with probability of 0.5. Equal numbers of subjects experienced schedule values of 15, 30 and 60 s, and for each schedule value, no-food trial durations were 8 times the schedule value for half of

subjects and 240 s for the rest. No-food trial duration had no systematic effect on response distributions, which were approximately normally distributed with peak times close to schedule values. Schedule value was positively correlated to start time, stop time, and the mean and standard deviation of time of response. Cheng and Westwood (1993) reported very high correspondence between start and stop times calculated in this way and those calculated based on their own criteria.

Zeiler and Powell (1994) further quantified pigeons' peak-procedure performance as functions of schedule value, using values ranging from 7.5 to 480 s. Latency to first response, start time and stop time were linear, linear and power (sensitivity >1) functions of schedule value, respectively. The coefficient of variation (CV) is a measure of relative variability: the standard deviation divided by the mean. In Zeiler and Powell's experiment, pause duration CV was an increasing function of schedule value, in violation of the scalar property, but 'middle' CV, based on midpoint between start and stop times, was constant. Zeiler and Powell's results show that different measures of temporal control involve different transformations of delay, but all can be indicators of temporal control.

1.5.4 Scalar expectancy theory and interval timing

The scalar hypothesis proposed by Gibbon (1971, 1972) set the stage for Scalar Expectancy Theory (SET; Gibbon, 1977), a predominant theory of timing. Gibbon (1971, 1972) reported that although some rodent subjects responded to avoid shock more efficiently than others, variability in response timing increased as shock density decreased for all subjects. Gibbon posited that measures of responding in time are determined by "estimates" of time to reinforcement or punishment. These estimates are scale transformations of a "unit timer" which tracks time elapsed from a procedurally relevant

time marker, such as a stimulus onset or reinforcer delivery. Later, Gibbon (1977) introduced the concept of 'expectancy' to provide a theoretical framework for explaining relationships between local response rate and conditional probability of reinforcement and between response variability and global reinforcement rate. According to SET, performance in interval schedules reflects the subject's estimate of how soon reinforcement will occur. Gibbon (1977, 1991) referred to this estimate as expectancy (denoted with *H* or *h*, for "hope").

According to SET, expectancy at the beginning of each trial in an interval (FI, VI, PI) schedule $h(0)$ is an estimate of "overall expectancy." Here, *V* denotes expectancy rather than Gibbon's *H* because it is equivalent to the concept of value in matching-based operant research as introduced by Baum and Rachlin (1969), scaled in proportion to the interval schedule value:

$$h(0) = V/d \ .$$

Equation 1.14

Typically used to refer to delay, *d* indicates the mean time to food experienced (or perceived) by the subject. After the beginning of each trial, expectancy increases hyperbolically as time since trial onset approaches expected time to reinforcement:

$$h(t) = V/(d - t) \ .$$

Equation 1.15

In Equation 1.15, *t* refers to time elapsed (or perceived time elapsed) in a trial. Because the expectancy function approaches infinity as expected time remaining to reinforcement approaches zero, it is not possible to obtain a closed-form expectancy function by integrating over the period from trial onset to scheduled reinforcement. SET resolves this

difficulty by estimating temporally mediated measures of behavior based upon expectancy attaining a threshold ratio:

> Responding emerges when a discriminable or "worthwhile" improvement in expectancy is achieved… As time in the interval elapses, subjects compare local expectancy value *h(t)*, with the estimate of the overall or undiscriminated expectancy of reward *h(0)*, by taking the ratio of local to overall expectancy, *h(t)/ h(0)*… Responding begins when this ratio exceeds a threshold value. (Gibbon, 1977, p. 282).

The critical assumption of SET is that temporally controlled changes in behavior within an interval occur when the ratio of instantaneous expectancy to average expectancy across the interval exceeds some threshold (assumed to be constant within an experimental setting). The important prediction of SET is that, because temporally controlled performance is determined by an expectancy ratio, its variability should be scalar – a constant proportion of delay. Gibbon (1977) showed that the assumption of scalar variability described performance in interval schedules better than assumptions that timing is determined by elicitation or by an absolute or Poisson process.

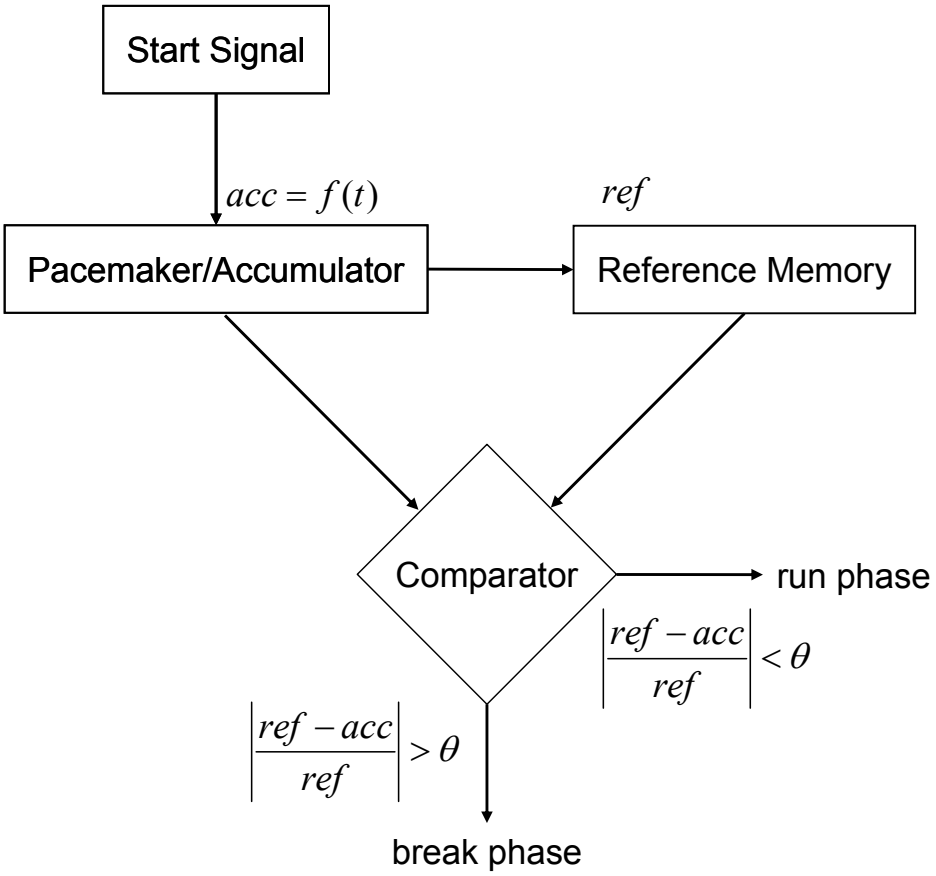

Figure 1.5. An information-processing model of timing (Church & Broadbent, 1990; Gibbon, 1991).

Figure 1.5 depicts an information-processing version of SET (Church & Broadbent, 1990; Gibbon, 1991; Church, Meck & Gibbon, 1994) that describes performance under FI schedules of reinforcement and in the peak procedure. Upon detection of a start signal, such as termination of reinforcer access or a stimulus change associated with trial onset, increments of a unit timer are collected in an accumulator, *acc*, until a subsequent event such as reinforcement resets it. Accumulator values represent relevant intervals experienced or perceived by the subject. Reference memory contains all previous accumulator values. Within a trial, time elapsed (as determined by accumulator value) is continuously compared to *ref*, an interval sampled from reference memory. Responding occurs at a high rate (the run phase) when the comparator ratio, ([absolute difference between *ref* and *acc*]/*ref*) is below a threshold, θ, and it occurs at a low rate

(break phase) when the ratio exceeds that threshold. The threshold may vary for different subjects and measures of temporal control, but SET assumes that it should be constant within an experimental setting.

The information-processing SET model describes the break-run pattern of responding in FI schedules and the break-run-break response pattern characteristic of the peak procedure: at the start of an interval, *ref* and *acc* are maximally dissimilar. At that point, the expectancy ratio, *h(t)/ h(0)* and comparator ratio, both equal 1. Between trial onset and expected time to food, expectancy ratio increases hyperbolically to infinity, the comparator ratio decreases monotonically to 0, and at some point the threshold is reached and the run phase begins. Under FI schedules, all trials should end with reinforcement during the run phase. In no-food trials of the peak procedure, expectancy ratio decreases and comparator ratio increases after expected time to reinforcement has elapsed. In the ideal, when the threshold is reached a second time, responding switches from a high rate back to a low rate and trials end without reinforcement during the second break phase.

Church, Meck and Gibbon (1994) applied SET to rats' responding in individual trials of the peak procedure. In addition to start and stop times, they calculated a middle (the midpoint between start and stop times) and spread of the run phase for each no-food trial and examined patterns of covariance between the four measures. They contended that serial (Killeen & Fetterman, 1988), quasi-serial (Gibbon & Church, 1992) and parallel scalar (i.e., SET) models of timing each should be associated with a distinct covariance pattern. Other than a negative correlation between start time and spread (which was anticipated; it indicates that when start times were relatively early, the run phase was relatively long), all correlations were positive. Church, Meck and Gibbon found that the

observed covariance pattern was consistent with predictions of SET but not with serial or quasi-serial predictions. Separate threshold parameters for predicting start and stop times produced sufficiently improved fits of variance and covariance to justify the additional parameter.

SET provides an explanatory framework for the break-run (-break) pattern of responding in FI (and PI) schedules: initiation (and termination) of the run phase is determined by the ratio of instantaneous to initial expectancy. The expectancy ratio also explains nondifferential responding over trial time elapsed observed in VI schedules (Stubbs & Pliskoff, 1962): because geometric progressions are memoryless, the (mathematically) expected time to food does not decrease as time spent in the interval increases, thus for all t, $h(t) = h(0) = V/d$. In addition to patterns of variance and covariance in interval schedules of reinforcement and punishment, SET provides a theoretical explanation of responding in other procedures including discrimination and choice tasks.

1.5.5 Scalar expectancy theory applied to choice behavior

SET has been extended to describe choice between delayed rewards. Gibbon (1977) proposed a means of predicting IRT distributions in the relatively uncommon concurrent differential reinforcement of low rates (DRL)-DRL procedure on the basis of an expectancy ratio discrimination rule. Gibbon extended this rule to steady-state choice between two delayed rewards, assuming that separate timers operate independently for each alternative. He predicted that the probability of choosing to make a "short" response should equal the area under the combined expectancy density function that is less than $b\beta S/C$, where $b\beta$ is determined by the thresholds associated with both alternatives and S and C are total time to reinforcement for the standard (or shorter) and comparison (longer) delays,

respectively. This expression is functionally equivalent to generalized matching in that variation in the efficiency of the unit timers can produce under- or overmatching. Further, Gibbon suggested that SET could be modified to account for observed preference for VI over FI terminal links (Killeen, 1968), and initial- and terminal-link effects could be explained by differences in expectancy ratios for the two alternatives.

Gibbon and Church (1981) developed the time-left procedure to study temporal discrimination. In their Experiment 1, once responding to FI 30 s ("standard') and FI 60 s ("comparison") schedules on separate levers had stabilized, four rats experienced combined trials in which only the comparison lever was available initially. The standard lever was introduced after 15, 30 or 45 s had elapsed in the trial. All subjects preferred the standard lever when it was introduced at 15 s and the comparison lever when the standard lever was introduced at 45 s. When the standard lever was introduced at 30 s and reinforcement became available on each lever at the same time, all subjects were approximately indifferent; equally likely to respond on the standard as the comparison lever. In their Experiment 2, three pigeons pecked in concurrent chains. A single VI schedule operated in initial-links, but terminal-link entries were not preselected; the first peck to either key once the VI schedule was satisfied would produce a terminal link. For the comparison terminal link, the delay to reinforcement was determined by the time spent in initial links, such that total time from initial-link onset to reinforcer delivery was constant. The delay for the standard terminal link was a fixed value. Subjects could maximize reinforcer immediacy by responding exclusively on the standard key until enough of the comparison interval had elapsed that reinforcement on the standard alternative would be scheduled later than reinforcement on the comparison alternative, thereafter responding exclusively on the comparison key. To a first approximation, pigeons' behavior was described by a

psychophysical function that suggested they were doing just that; indifference points occurred when delay to reinforcement on standard and comparison alternatives was the same. Gibbon and Church concluded that pigeons' subjective time was linear (not logarithmic) in real time. They reasoned that SET could predict the results of their Experiments 1 and 2 by incorporating the assumption that time is represented linearly.

Gibbon, Church, Fairhurst and Kacelnik (1988) proposed memory and decision mechanisms to explain matching in concurrent VI-VI schedules, overmatching in concurrent chains with fixed-duration terminal links, and preference for variable over fixed terminal links. For choice between fixed reward delays, Gibbon et al. assumed that subjects held in memory a normally distributed representation of each delay, with mean linearly proportional to the real-time delay and scalar variance. The subject estimates subjective time to food by sampling single values from the memorial representation for each alternative, and compares the ratio of the samples to a criterion value. The subject then responds to the alternative associated with the shorter sampled delay. Therefore, predicted preference for the more immediate terminal-link equals the probability that, for any two values sampled at random from their respective memorial representations, the shorter of the two samples was drawn from the representation of the shorter delay. With scalar variance in memory, predicted preference for the shorter terminal link is more extreme than the immediacy ratio; overmatching is correctly predicted.

Gibbon et al. (1988) extended their theory of choice to account for matching and undermatching in concurrent VI-VI schedules. They assumed that each alternative is associated with a separate memorial representation, but that all delays experienced in association with an alternative were remembered with scalar variability and contributed to

the representation. Gibbon et al. assumed exponential distributions with means equal to the VI schedule value were adequate approximations of the resultant distributions. Gibbon et al. predicted that pigeons would choose the alternative associated with the shorter sampled time to food, with samples drawn from the separate representations. This version of SET predicts matching if there is no bias and preference for variable over fixed delays. Gibbon et al. (1988) showed that SET can predict overmatching when delays are fixed, and preference for variable over fixed delays using the time-left procedure. They argued that no contemporary theories of choice based on the matching law (including DRT, Squires & Fantino, 1971) could account for the observed empirical phenomena. However, Preston (1994) noted that when Gibbon et al. (1988) calculated time-to-food for DRT, GML and Incentive theory predictions, they included initial-link time for the comparison but not the standard alternative. When Preston (1994) recalculated predictions using only terminal-link delays, matching-based theories produced comparable accounts of performance in the time-left procedure.

Gibbon (1977) and colleagues (Gibbon & Church, 1981; Gibbon et al., 1988) have extended SET to describe choice between two reinforcement alternatives. This extension relies on assumptions about representation of delays in memory. Nevertheless, SET is a theory of timing that describes choice between fixed and variable delayed rewards.

1.6 Timing in transition

The same advantages to studying choice in transition apply to timing in transition. There is ecological validity in studying changes in interval duration. Many and frequent changes in interval duration can illustrate functional relationships between interval duration and temporal control. Investigation of temporal dynamics is a critical element in the

empirical determination of the process generating temporal control behavior. Relative to concurrent-chains choice in transition, there has been an abundance of empirical and theoretical work on interval timing in transition.

1.6.1 Acquisition and scalar expectancy theory

Early mathematical models of learning (e.g., Bush & Mosteller, 1955; Estes, 1950; Rescorla & Wagner, 1972) employed linear-operator or associative principles. They assumed that when contingencies changed, behavior would adjust gradually. Empirical results of peak procedure studies suggest that, just as responding within trials switches abruptly from a low rate in the break phase to a high rate in the run phase, behavioral adjustments to new schedule values are abrupt. Meck, Komeily-Zadeh and Church (1984) examined the effects transferring rats from PI 10 s to PI 20 s and PI 20 s to PI 10 s schedules. They plotted peak time as a function of number of trials since schedule value change. Changes in peak time were two-step functions, both when the schedule value was increased and when it was decreased. They hypothesized that rats employed a ratio of the representations of old and new durations in reference memory to arrive at an intermediate peak time, which perseverated until rats' performance was no longer influenced by the previous schedule value.

Lejeune, Ferrara, Simons and Wearden (1997) trained pigeons in a peak procedure. Schedule value was either PI 10, 20, 30 or 40 s and it changed systematically (increased then decreased or vice versa), either every session or every 3 sessions. Like Meck, Komiely-Zadeh and Church (1984), Lejeune et al. also found that adjustments of peak times to a new schedule value occurred rapidly and abruptly when schedule value was increased. However, Lejeune et al. (1997, p. 211) reported that when schedule value was

decreased, peak-time transitions were "mostly smooth but rapid." They pointed out that subjects experienced nonreinforcement at the previous schedule value when schedule value was increased but not when it was decreased. Lejeune et al. suggested that proactive interference by the previous schedule in the latter case accounted for the asymmetry in peak time changes.

1.6.2 Rate estimation theory

Although Meck, Komeily-Zadeh and Church's (1984) and Lejeune et al.'s (1997) results were consistent with the assumption of SET that responding in the peak procedure reflects a frequently updated representation of delay, at its core SET is a steady-state model for timing and it does not predict behavior under dynamic conditions. Gallistel and Gibbon (2000) presented rate estimation theory (RET) as a unified theoretical approach to explaining patterns of behavior under a variety of conditions. It is functionally related to SET but developed specifically as a non-associative account of Pavlovian acquisition and extinction behavior. A distinctive feature of RET is that is proposes a common quantitative and conceptual foundation of both operant and classical learning. Separate but related decision processes govern whether and when a subject responds. Both the *whether* and *when* decision processes involve ratio comparisons of the current situation to the subject's recent reinforcement history. The *when* decision process is essentially an extension of SET applied to acquisition and extinction, involving representations in working and reference memory of durations relative to a particular time marker determined via a pacemaker-accumulator. The conceptual foundation of RET makes possible quantitative predictions about a range of behavioral phenomena, including temporal control in the peak procedure and response allocation in concurrent procedures.

In addition to the pacemaker-accumulator and reference memory, RET and SET share the feature of a comparator with a ratio threshold assumed to be constant. Decisions to start and stop responding at a high rate in the peak procedure occur as depicted in Figure 1.5 and described in Section 1.5.4. Thus, RET makes similar predictions to SET about temporal control, and similarly accounts for scalar variability and timescale invariance. To describe acquisition of temporal control, Gallistel and Gibbon (2000) incorporated two additional principles. First, RET assumes that representation of relevant time intervals cumulate across trials and sessions as long as reinforcement rate does not change. RET also incorporates a partitioning process that compares rate of reinforcement in the presence of a conditioned stimulus with a baseline or 'background' rate of reinforcement applied to ITIs. In FI schedules of reinforcement, this background reinforcement rate is assumed to equal the overall rate of reinforcement initially. Gallistel and Gibbon's rationale for this assumption is that "when only a few reinforcements have occurred in the presence of a CS, it is inherently ambiguous whether they should be credited entirely to the CS, entirely to the background, or some to each." (p. 302). Partitioning occurs over the course of a session; subjective cumulated time in ITIs increases and therefore background reinforcement rate decreases. A prediction of this partitioning principle is that the number of reinforcers a subject experiences before responding stabilizes is inversely proportional to the ratio of ITI to FI schedule value. Gallistel and Gibbon reported a reanalysis of 12 delay-conditioning data sets that showed a negative linear relationship between reinforcements to acquisition and ITI to CS presentation duration. They concluded that this prediction was approximately correct. In simple classical conditioning paradigms (and, by extension, simple operant schedules of reinforcement), RET correctly predicts effects of trial spacing and delay to reinforcement, no effects of partial reinforcement or reinforcer magnitude,

timescale invariance, and the scalar property. Applied to timing behavior, RET represents a successful extension of SET to acquisition and extinction.

To extend RET to the analysis of operant choice, Gallistel and Gibbon (2000) incorporated the idea that different decision variables determine different aspects of learned behavior. They proposed two decision mechanisms, opting and allocating, each applied when it is adaptive to do so. The two-mechanism process of RET distinguishes it from theories of choice developed to account for matching behavior (including DRT, CCM and HVA). Although such models vary in the assumptions and predictions they make, a fundamental underlying principle they share is that the matching result is due to a fixed, monotonic correspondence between the relative subjective values of two options and measures of preference.

The opting mechanism Gallistel and Gibbon (2000) described is a winner-take-all process that optimizes obtained reinforcement by always selecting the 'best' option. Gallistel and Gibbon assumed that opting occurs whenever it is not possible for the subject to exploit both (or all) alternatives concurrently (as is the case in concurrent chains). The opting mechanism is similar to the process described by Gibbon et al. (1988); in memory, there is a representation of delay for each alternative. The subject samples from each representation repeatedly during initial links and always responds to the alternative associated with the shorter sampled delay. Gallistel and Gibbon assumed that non-exclusive choice occurs because occasionally the delay sampled from the longer representation is shorter than the one sampled from the shorter representation. At the molar level, response allocation should equal expected relative immediacies (not relative expected immediacies). Functionally, opting is similar to hyperbolic discounting, which makes it

possible for RET to incorporate effects of reinforcer magnitude. Although Gallistel and Gibbon did not address effects of absolute initial or terminal-link duration, the opting process of RET provides a promising alternative framework for predicting concurrent-chains choice.

Gallistel and Gibbon (2000) proposed that an allocating mechanism applies when primary reinforcers are concurrently available on multiple alternatives, as is the case in concurrent schedules. Like melioration (Myerson and Miezen, 1980), allocating is a process that maximizes net gain. In two-key concurrent schedules, the allocating mechanism determines the point at which the momentary probability of reinforcement is greater on the opposite alternative than it is on the one the subject is currently responding to and dictates that the subject change over to the other key at that point. The relevant dependent variable is not relative rates or numbers of responses but leaving rates. RET predicts visit termination on the basis of three empirically-supported assumptions: 1) leaving is stochastic 2) overall leaving rate is an increasing function of overall reinforcement, and 3) the relative duration of stays is proportional to relative magnitude of the reinforcer. Although there are a number of features of concurrent schedule and concurrent-chains choice that Gallistel and Gibbon (2000) did not address, the opting and allocating processes of RET provide an alternative framework for predicting concurrent-chains choice, making RET an appealingly comprehensive account of behavior.

1.6.3 The linear waiting hypothesis and "the ability to track"

The 'one-back' linear waiting hypothesis (LW) put forward by Staddon, Wynne and Higa (1991) is a dynamic timing model meant to address the "lack of a mechanistic theory for schedule effects." LW quantifies Wynne and Staddon's (1988) observation that

wait time (latency to first response, their measure of temporal discrimination) is a linear function of the most recently experienced interval between a time marker and food. Any event with a reliable and unambiguously identifiable temporal relationship to reinforcement can serve as a time marker. Stimulus changes, reinforcer deliveries and (in some cases) responses can serve as time markers. Staddon et al. (1991) proposed that a quasidynamic process determines wait time, *t*:

$$\delta(t) = G[AI(N) + B - t(N)].$$

Equation 1.16

Equation 1.16 (their Equation 3), $\delta(t)$ is the change in *t* from the *N*th interfood interval (IFI_N) to IFI_{N+1}, $t(N)$ is the interval between the time marker and the response (i.e., the wait time) and $I(N)$ is the interval between the time marker and food in IFI_N. A is a constant less than one, and B is small. G is some positive monotonic function that goes through the origin. Staddon et al. assumed for simplicity's sake that $G(X) = X$, in which case, wait time in IFI_{N+1} is a linear function of IFI_N:

$$t(N+1) = AI(N) + B .$$

Equation 1.17

LW can predict the acquisition trajectory of post-reinforcement pausing and the "scallop," or pause at the beginning of FI schedules of reinforcement: once wait time is a function of the IFI, the first post-reinforcement response becomes a time marker, increasing wait time on successive IFIs until it reaches equilibrium. It also can predict several steady-state features of responding in VI schedules and the rapid development of a high rate of responding on ratio schedules. However, LW is not able to account for the difference in response rate for interval and ratio schedules equated for rate of reinforcement.

Higa, Wynne and Staddon (1991) reported three experiments in which pigeons experienced sinusoidal sequences of IFIs in a response-initiated delay (RID) procedure. The first peck of each IFI changed the color of the key from red to green; subsequent pecks had no effect. Higa, Wynne and Staddon computed product-moment correlations for wait time and schedule value from the Lag 0-4 IFIs for each subject and condition. With one exception, Lag 0 or Lag 1 always had the highest correlation with schedule value, which indicated that wait times were determined by the current or previous IFI. Whereas pigeons were able to "track" sinusoidal sequences of IFIs, pigeons presented with a square-wave sequence of IFIs (Staddon, 1967) failed to discriminate between short and long IFIs, even though the discrimination should have been "simpler". LW predicts that temporal discrimination should track all changes in IFI equally well. Higa, Thaw & Staddon (1993) reported two experiments investigating pigeons' performance on RID schedules with 'square wave' changes to the IFI interval. They exposed pigeons to RID schedules of 5, 15 and 45 s. When the schedule value changed only once in a session, wait times increased and stabilized within 2-3 trials when the new delay was longer than the old. When the new delay was shorter than the old, wait times decreased from the second post-change IFI, but stabilization took longer than when the delay increased. Higa (1997) confirmed that effects of previous delays on wait time were more enduring when delays decreased than when they increased.

Staddon and Higa (1999) expanded on LW to propose a testable theory of timing designed as a pacemaker-free alternative to SET. Their multiple-time-scale (MTS) model for timing was based on the principle of rate-sensitive habituation. Like SET, MTS assumed separate estimates of the current time and memory of previously-experienced intervals between a time marker and reinforcement. It also assumed that the process

generating interval timing-related behavior involved a comparison between current and remembered intervals. The principal difference between MTS and SET is the process assumed to generate current and remembered intervals. SET and related theories assume a pacemaker-accumulator process determines current and remembered intervals. This process describes steady-state behavior but temporal dynamics are beyond its scope. By contrast, Staddon and Higa proposed that an intrinsically dynamic, concatenated memory trace process determines interval representations. They showed how, in principle, MTS can account for a wide variety of time-related behavioral phenomena including timescale invariance and the scalar property, without reference to a pacemaker-accumulator. They did not include closed-form expressions for MTS, precluding quantitative comparisons with other models of timing.

One-back LW is an appealing hypothesis because it is simple, testable and mechanistic (rather than functional). It is able to explain some of the same response properties as expectancy models including SET (Gibbon, 1977) no assumptions about internal memories. Higa and colleagues (Higa, 1997; Higa, Thaw & Staddon, 1993; Higa & Tillou, 2001; Higa, Wynne & Staddon, 1991) demonstrated that pigeons' wait times can track cyclic changes in IFI. However, observed asymmetries in performance when IFIs increased as opposed to decreased, apparent effects of previous IFIs and the failure to track observed in some circumstances are not consistent with LW. Ultimately, Higa, Thaw & Staddon (1993, p. 541) rejected 'one-back' LW as "only an approximation of a mechanism, working under a limited set of conditions." Nevertheless, Staddon, Wynne and Higa's (1991) idea that intervals between individual time markers and food have cumulative effects on temporal discrimination may prove to be important.

1.7 Integrated choice and timing research

Both choice and timing have been prominent topics of study and both are understood to be critical elements of a science of behavior. However, they typically have been studied in isolation by researchers with different theoretical orientations. Theoretical accounts of behavior that address both choice and timing (Gallistel & Gibbon, 2000; Staddon & Cerutti, 2003) describe foundational behavioral principles and assumptions that might apply to responding in choice as well as timing procedures. In spite of the separate extensive bodies of literature in the experimental analysis of behavior on choice and timing, there have been few empirical attempts at integration.

What constitutes an integrated study of choice and timing? The empirical reports that have been published all use a method of converging operations. Garner, Hake and Eriksen (1956, p. 150) described converging operations as "any set of two or more experimental operations which allow the selection or elimination of alternative hypotheses or concepts which could explain an experimental result." For information about choice and timing to be considered convergent, behavioral measures of each must be collected and analyzed at the same level (Skinner, 1950).

Grace and Nevin (1999) investigated the role of timing processes in choice by embedding a peak procedure into a concurrent-chains schedule. In their experiment, pigeons received baseline training in a multiple-peak interval schedule in which PI 40 s and PI 20 s (or PI 20 s and PI 10 s) trials were differentially signaled by lighting the center key of a three-key operant chamber red or green. No-food trials ended after 60 s. During that time, pecks had no effect. After 25 baseline sessions the 15-s ITI was replaced with a VI 8-s schedule that operated concurrently on the two side keys, which served as initial links.

The left or right response that satisfied the initial-link schedule produced a terminal link analogous to one of the PI schedules that had operated during baseline. Response distributions from no-food trials, peak medians and interquartile ranges, all measures of temporal control, were comparable in the two conditions. Subjects demonstrated a strong preference for the initial link leading to the shorter terminal-link delay, consistent with the overmatching observed in concurrent chains with FI terminal links in which all trials end in reinforcement (Omino & Ito, 1993).

Pigeons then received a replication of the two conditions in which the PI 40-s schedule was replaced with a PI 10-s schedule, or vice versa. In the multiple peak interval condition, location of peak response rate on no-food trials quickly adapted to the new schedule value. Upon reinstatement of the concurrent-chain condition, adjustment of preference to the new immediacy ratio (which was the reciprocal of what it had been in the first concurrent-chains condition) took place over multiple sessions for all subjects. Critically, at the beginning of the final condition, pigeons' response allocation favored the longer terminal link even though terminal-link responding showed evidence for accurate temporal control. Grace and Nevin concluded that the rapid adjustment of temporal control and gradual adjustment of initial-link response allocation represented a dissociation of timing and choice processes and posed a challenge to theories of choice based on an underlying timing process.

Using a successive-reversals design and a concurrent-chains plus peak procedure, Grace (2002b) examined acquisition of preference and of temporal control. Responding in initial links produced terminal links according to a single VI schedule. Terminal links were PI 10 and 20 s in one experiment and PI 8 and 16 s in the other, and the left:right location of

each schedule changed several times after responding had stabilized. The primary dependent measures of preference and temporal control, respectively, were initial-link response ratios and relative expected immediacies. Grace calculated "relative expected immediacy" to reinforcement by taking the ratio of the reciprocals of peak times for left and right alternatives. To compare preference and temporal control across successive reversals, Grace calculated mean initial-link response ratios and relative expected immediacies for each subject, aggregating data over the last 5 sessions of each condition. Both adapted to reversals in terminal-link schedules, consistent with previous research. Relative expected immediacy approximately matched relative programmed immediacy and initial-link response ratios overmatched programmed immediacy ratios. To compare acquisition of preference and temporal control, Grace calculated initial-link response ratios and relative expected immediacies for individual sessions. Across sessions, relative expected immediacy adjusted to successive reversals more rapidly than did response ratios. Grace (2002b, p. 32) regarded this as positive evidence that concurrent-chains choice is determined by "learned value" of terminal-link stimuli, and concluded by opining that "an important goal for future research will be to explore relations between initial- and terminal-link responding at the level of individual trials, in an attempt to understand better how choice is mediated by value."

Cerutti and Staddon (2004) examined the relationship between behavioral measures of temporal control and control by rate of reinforcement in concurrent chains, multiple and simple concurrent schedules. In their Experiment 1, Cerutti and Staddon trained pigeons in concurrent chains with independent initial links. Terminal-link entry was programmed to occur with a probability of 0.025 every second and was not contingent on responding. The two terminal-link delays were always FI 15 s and FI 45 s. The left:right

location of the shorter terminal-link delay reversed mid-session after varying numbers of sessions. They found that initial-link response ratio was highly positively correlated with the ratio of left initial-link response latency to right initial-link response latency. Within individual sessions, relative initial-link response rate was a power function of relative initial link latencies and relative terminal-link response rates covaried with relative terminal-link latencies, although there was little overall variability in terminal-link response rate or latency across the two FI schedules. Post-transition adjustments to different terminal-link delays were more rapid in terminal link than initial link measures of behavior. The major result of all three experiments is that several properties of initial- and terminal-link responding covary with terminal-link delays, and have similar acquisition trajectories to response allocation. Staddon and Cerutti called for greater attention to temporal measures of behavior in free-operant choice experiments.

Based on investigations of pigeons' timing performance in modified concurrent schedule procedures, Jozefowiez, Cerutti and Staddon (2005, 2006) proposed that independent processes determine temporal patterns and allocation of responding in two-key choice procedures. In one (2005) experiment, they compared performance in a concurrent schedule in which intervals for both schedules reset upon reinforcement of either schedule with performance in which the same schedules operated concurrently and independently. Pigeons' overall rates and temporal patterns of responding to variable and fixed schedules differed in the resetting procedure but not in the non-resetting procedure, and when responses in the resetting procedure were combined across keys, temporal pattern was unaffected by reinforcer ratio. In a second experiment, two keys were reinforced probabilistically with the same IFI but different FI durations. Pigeons responded more to the alternative associated with the shorter delay early in the trial and more to the alternative

associated with the longer delay later in the trial. Jozefowiez, Cerutti and Staddon (2005) concluded that pigeons timed trial duration, not IFI. They suggested that the invariance of combined responding in the resetting procedure and responding on each key in the non-resetting procedure implied separate timing and allocation processes.

Jozefowiez, Cerutti and Staddon (2006) reported a second set of experiments in which pigeons were trained in concurrent mixed-interval schedules. Pigeons' overall response rate (based on responses from both alternatives) was timescale invariant as a function of absolute schedule duration. Relative response rate favored the shorter schedule more when delays were overall short. At the molar level, this discrepancy between 'overall' and 'relative' responding is consistent with effects of overall rate of reinforcement on sensitivity in concurrent VI-VI schedules reported by Alsop and Elliffe (1988). However, Jozefowiez, Cerutti and Staddon also reported timescale invariance of indifference points, suggesting Weber's law can apply to both total and relative responding in concurrent-schedule procedures. They reasoned that these findings were consistent with their previously presented independent-processes hypothesis.

In another successive-reversals concurrent-chains experiment, Berg and Grace (2006) investigated effects of initial-link duration on choice and temporal control in transition. The location of the shorter of two terminal link schedule values switched every 20 sessions and initial links were either relatively long (VI 24 s) or short (VI 8 s). They replicated the initial-link effect: preference for the shorter terminal link was less extreme when the initial-link schedule was long. The magnitude of change in response allocation over the first three post-reversal sessions was greater when initial-link duration was short before the reversal and long after it, and smaller when initial-link duration was long before

the reversal and short after it. Consistent with Jozefowiez, Cerutti and Staddon's (2005) conclusion that pigeons time trial durations rather than IFIs, initial-link duration had no effect on terminal-link responding. Berg and Grace contended these results were consistent with the theoretical assumptions of DRT.

Grace, Berg and Kyonka (2006) also investigated effects of initial-link duration on initial-link response allocation as well as start and stop times and response distributions from no-food terminal links in a steady-state concurrent chains plus peak procedure. Terminal-link schedules were FI 8 s and FI 16 s, and the VI initial-link schedule was either long (24 s) or short (8 s). Pigeons' preference for the FI 8 s terminal link was greater when initial links were short than when they were long, replicating the initial-link effect. There was no effect of initial-link duration on start or stop times. Regression analyses showed that local measures of initial-link responding did not account for variability in measures of timing beyond variability explained by the FI schedule. Grace et al. (p. 199) concluded that there was no evidence of interdependence between measures of choice and temporal control, or of mediation by a "common representation of delay."

The seven papers discussed in this section include concurrent-schedule and concurrent-chain experiments. Three involved successive-reversal designs, and although Grace and Nevin (1999) presented a steady-state experiment, their most interesting and novel result was the dissociation in acquisition trajectories of preference and temporal control. Taken together, it is clear that choice and of temporal control can be dissociated. Do these results imply that separate, independent processes determine response generation and response allocation, as Jozefowiez, Cerutti and Staddon (2005, 2006) hypothesized? Evidence that acquisition of temporal control precedes acquisition of preference (Cerutti &

Staddon, 2004; Grace, 2002; Grace & Nevin, 1999) could indicate separate processes, or it could imply primacy of temporal control, that temporal learning drives preference. The respective dependence and independence of initial- and terminal-link responding on initial-link duration (Berg & Grace, 2006; Grace, Berg & Kyonka, 2006) indicate that nontemporal factors can control acquisition of preference in concurrent chains. However, the presence of other predictive factors does not eliminate the possibility of temporal control. Rather than supporting the hypothesis that independent processes govern temporal control and choice behavior, results of concurrent-chains experiments suggest that measures of temporal control may have predictive utility beyond that attributable to delay alone in predicting choice between delayed rewards, particularly in non-steady-state procedures.

1.8 A quasidynamic decision model for choice between delayed rewards

Grace and McLean (2006) proposed a quasidynamic (Staddon, 1988) linear-operator model for concurrent-chains choice between delayed rewards. Their "decision model" estimates (mathematically) expected initial-link response strength for each alternative based on comparison to a shared comparison distribution. It assumes that each time an organism experiences a terminal link, response strength increases if the terminal-link delay is categorized 'short' relative to the criterion and decreases if it is categorized 'long' relative to the criterion. The decision model predicts that at any point in a session (after n reinforcers or blocks of reinforcers) relative expected response strength, RS, determines response allocation:

$$\log\left(\frac{B_{Ln}}{B_{Rn}}\right) = \log\left(\frac{RS_{Ln}}{RS_{Rn}}\right).$$

Equation 1.18

Each time the organism experiences a terminal link, expected response strength for that alternative increases in proportion to the probability the terminal-link was categorized 'short' and decreases in proportion to the probability it was categorized 'long:'

$$RS_{n+1} = RS_n + p_{short} * (Max_{RS} - RS_n) * \Delta - (1 - p_{short}) * (RS_n - Min_{RS}) * \Delta .$$

Equation 1.19

Expected response strength for an alternative in initial links after the subject has experienced that terminal link *n* times (or *n* blocks of times), RS_{n+1}, is determined by RS_n, previous response strength, Max_{RS} and Min_{RS}, maximum and minimum possible response strengths, p_{short}, the probability that the just-experienced terminal-link delay was categorized short, and Δ, a learning rate parameter. The probability of a 'long' categorization is (1- p_{short}). The value of p_{short} is calculated as the probability that the programmed log terminal link delay is less than a randomly-selected log delay from a normal distribution specified by a mean, referred to as the criterion, and a standard deviation, σ. The same criterion determines probabilities for both terminal links. Grace and McLean used logarithmic, not linear delays in their calculations.

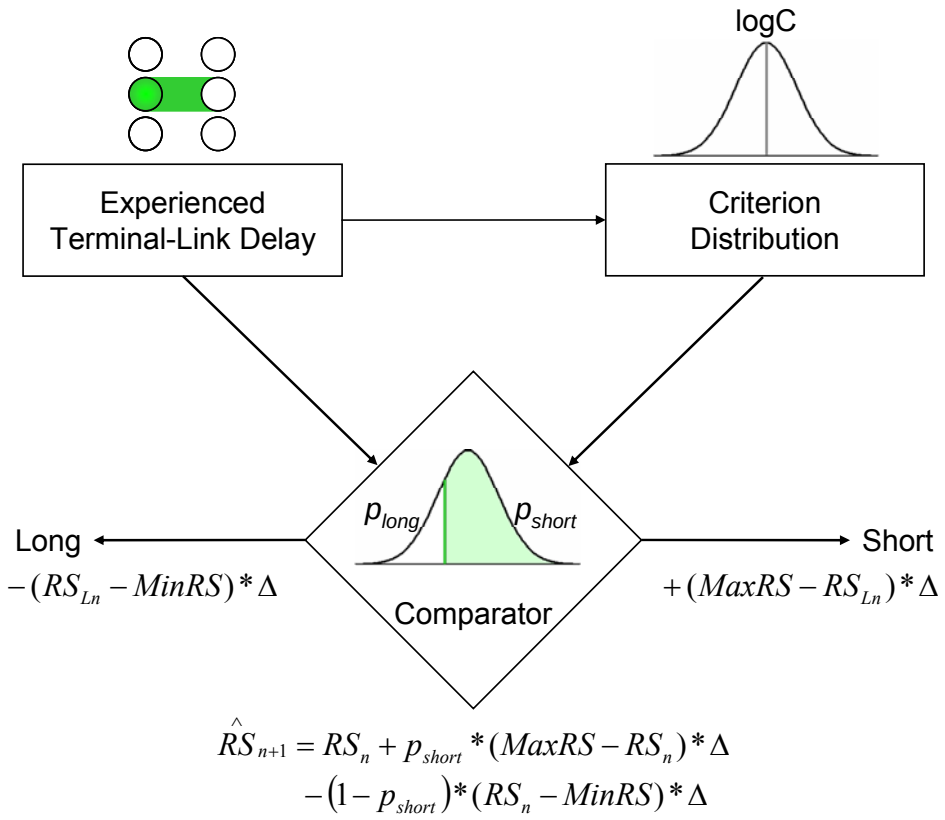

Figure 1.6. A decision model for concurrent-chains choice (Grace & McLean, 2006).

Figure 1.6 depicts an information-processing version of the decision model. Suppose a pigeon experiences a right terminal link that is reinforced according to an FI schedule. The logarithm of the interval between terminal-link onset and reinforcement is compared to the criterion. Response strength is only affected for the terminal link that was presented, so left response strength does not change. Right response strength would increase if the interval was categorized short and decrease if it was categorized long. The probability of the interval being categorized short or long, respectively, is equal to the filled and unfilled proportions of the criterion distribution. Expected response strength increases or decreases probabilistically. If the assumptions of the decision model are correct, expected response strength will only equal 'actual' response strength when p_{short} is correctly estimated as 0 or 1. However, relative expected response strength should predict response

allocation aggregated over a few initial links making it possible to construct within-session acquisition trajectories.

Effects of individual fitted parameters in the decision model are interdependent to some degree and not easily described. However, when minimum and maximum response strength are fixed, some phenomena are attributable to specific parameters. The learning-rate parameter, Δ, determines how quickly response allocation stabilizes. The σ parameter determines the likelihood subjects will categorize delays veridically, with smaller values producing categorizations that are more accurate. The relationship between predicted log response ratio and log immediacy ratio is approximately linear if σ is large and sigmoidal if it is small. Grace and McLean (2006) successfully applied the decision model to response allocation data that was typical of steady-state performance (i.e., linear) and to nonlinear data representative of performance in rapid acquisition procedures in which immediacy ratios changed unpredictably across sessions.

Christensen and Grace (2008) exposed pigeons to a rapid acquisition concurrent chains procedure in which initial-link schedule value systematically increased and decreased across sessions between VI 0.01 s and VI 30 s. Response allocation adjusted to unpredictable changes in immediacy ratio. However, changing initial-link durations affected pigeons' adjustments. Initial-link preference was more extreme in the ascending than the descending part of the sequence. In addition, the effect of initial-link duration on response allocation was bitonic: sensitivity to log immediacy ratios increased as programmed initial-link duration increased from 0.01 to 7.5 s, then decreased as initial link duration increased to 30 s. In Christensen and Grace's extended version of the decision model, response strength did not reset to a neutral value but only partially reset. This

carryover of response strength from previous sessions predicted the discrepancy in preference between ascending and descending parts of the sequence. Christensen and Grace also predicted the bitonic initial-link effect by assuming that the comparison distribution represented the history of initial- and terminal-link delays between stimuli experienced by the subject. In experiments in which absolute terminal-link duration increased and decreased systematically, Christensen and Grace (2009a, b) showed that the same additional assumptions also predicted the discrepancy in preference on ascending and descending parts of that sequence, as well as the terminal-link effect.

1.9 Objectives of the present research

The five concurrent-chains experiments and one concurrent-schedule experiment presented in this thesis are rapid acquisition procedures in which the contingencies arranged on each of two keys changed unpredictably across sessions. In Experiments 1-3, pigeons responded in concurrent chains plus peak procedures. Start and stop times were collected from occasional no-food terminal links. There were two conditions in Experiment 1, minimal and maximal variation. In the minimal-variation condition, terminal-link schedules were always FI 10 s and FI 20 s. The location of the shorter delay changed unpredictably across sessions. The maximal-variation condition was similar to the minimal-variation condition, except that each session, delay pairs were sampled from a potentially infinite population of values. The major objectives of Experiment 1 were to characterize acquisition trajectories and relationships between initial-link response allocation and relative immediacy and between measures of temporal control and terminal-link delay. An important additional goal was to determine whether there was covariation in measures of choice and timing beyond what could be attributed to separate functional

relations to delay. Experiments 2 and 3 investigated initial and terminal-link effects in rapid acquisition concurrent chains procedures. Experiment 2 was a minimal-variation experiment in which initial-link duration changed pseudorandomly each session. Experiment 3 was a maximal-variation experiment in which terminal-link delays were either overall short or overall long.

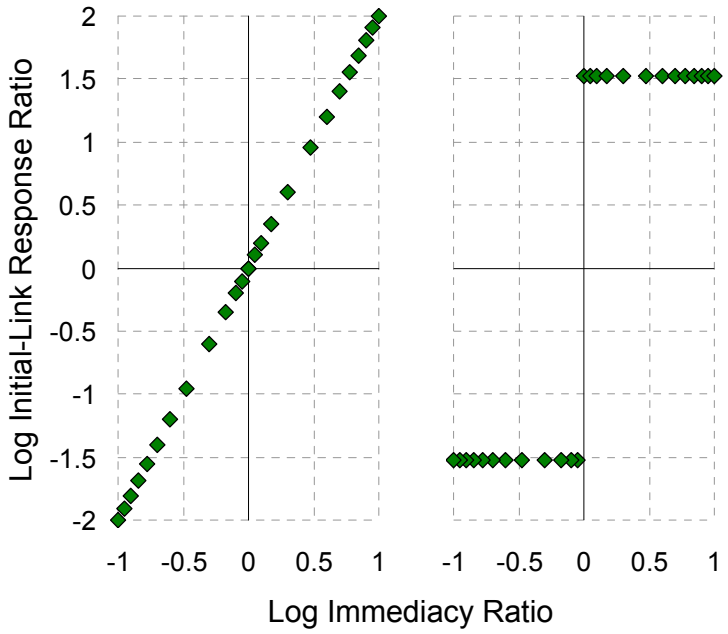

Figure 1.7. Two types of overmatching.

The underlying assumptions of different concurrent-chains choice models make different predictions about the processes that generate response allocation. Both processes produce overmatching. Steady-state theories such as DRT, CCM and HVA assume a matching mechanism in which initial-link responding corresponds to relative value. They predict a linear relationship between log response ratios and log immediacy ratios like the one depicted in the right panel of Figure 1.7. Expectancy theories of choice (extensions of SET, RET and other timing models) assume an opting mechanism in which deviations from categorical responding, like that pictured in the right panel of Figure 1.7, are due to errors in delay discrimination. RET also predicts covariation of choice and temporal control,

categorical preference, and abrupt, rather than gradual, adjustments to arranged contingencies.

Experiments 4-6 represent alternative rapid acquisition procedures investigating how pigeons adapt to frequent, complex changes in contingencies of reinforcement. Experiments 4 and 5 are concurrent chains in which relative immediacy and relative magnitude (and relative probability, in Experiment 5) changed according to independent random sequences across sessions. In Experiment 6, pigeons responded in concurrent schedules in which the rate and magnitude of reinforcement arranged on two keys changed independently and randomly across sessions. The goals of Experiments 4-6 were to determine whether acquisition of preference and stable levels of responding were determined by more than one dimension. Would response allocation adapt, and if so, how rapidly and how much?

Finally, the decision model proposed by Grace and McLean (2006) can account for phenomena observed in steady-state and rapid acquisition experiments. In Chapter 6, adapted versions of the decision model are applied to response allocation results of Experiments 1-6.

2 Chapter 2 Rapid acquisition of choice and timing: converging operations

2.1 Notes on Experiment 1

I published Experiment 1 with my supervisor Randolph Grace as "Rapid acquisition of choice and timing in pigeons," in the *Journal of Experimental Psychology: Animal Behavior Processes, 23*(4), in 2007. Citations of Kyonka and Grace (2007) in Chapters 3-6 refer to this article. It was the first time the rapid acquisition approach had been applied to a concurrent-chains plus peak procedure. Our goals were to characterize and compare initial-link response allocation, start times and stop times in minimal- and maximal-variation conditions, and to determine whether the procedure would yield results worth pursuing. We decided to present analyses of residual covariation in relative initial-link responding and terminal-link temporal control in terms of an hypothesis proposing that a 'common representation of delay' governed the acquisition of preference and of temporal control. We reasoned that, if choice and relative timing are correlated after their respective relationships to relative immediacy had been accounted for, the processes that generated choice and timing could not be wholly independent.

2.2 Experiment 1: Rapid acquisition of choice and timing in pigeons

2.2.1 Abstract

Pigeons were trained on a concurrent-chains procedure in which the initial link associated with the shorter terminal-link delay to food changed unpredictably across sessions. In the minimal-variation condition, delays were always 10 s and 20 s, whereas in the maximal-variation condition delays were generated pseudorandomly for each session. On some terminal links, food was withheld to obtain measures of temporal control. Measures of choice (log initial-link response ratios) and timing (start and stop times on no-food trials) showed temporal control and stabilized within the first half of each session. In the maximal-variation condition, choice was a nonlinear function of the log delay ratio, consistent with a categorical discrimination but contrary to models based on the matching law. Residuals from separate regressions of log response and log start and stop time ratios on log delay ratios were positively correlated. Overall, results support cognitive models which assume that initial-link choice is based on an 'all or none' decision process, and that choice and timing are mediated by a common representation of delay.

Keywords: choice, timing, concurrent chains, temporal dynamics, acquisition, key peck, pigeons.

2.2.2 Introduction

Choice between delayed rewards has been extensively studied using behavioral procedures such as concurrent chains. In a typical version of this task with pigeons, each trial starts with a choice phase or initial link during which subjects respond by pecking at either of two illuminated keys in an experimental chamber. Responses during the initial link produce one of two mutually-exclusive outcomes or terminal links, which end with food reinforcement after a delay has elapsed. The usual result, reported in many studies and replicated with other species such as rats and humans, is that subjects respond more to the initial link preceding the terminal link associated with the relatively shorter delay to reinforcement (see Mazur, 2001, for review).

Although the empirical results are well established, there remains substantial debate regarding the behavioral and cognitive processes underlying choice between delayed rewards. One major class of models for concurrent chains is based on the matching law and invokes conditioned reinforcement as a theoretical construct. In its generalized form, the matching law states that response allocation in concurrent variable-interval (VI) VI schedules is a power function of relative reinforcer rate (Baum, 1974):

$$\frac{B_L}{B_R} = b\left(\frac{R_L}{R_R}\right)^a,$$

Equation 2.1

where B is response rate, R is reinforcer rate, and the subscripts L and R indicate the alternative schedules. Equation 2.1 has two parameters: bias, b, which is a constant preference for responding to either alternative that is independent of the reinforcer ratio, and an exponent, a, which is the sensitivity of response allocation to the reinforcer ratio.

Typically, a logarithmic version of Equation 2.1 is used in which the log response ratio is a linear function of the log reinforcer ratio, with *a* and log *b* as slope and intercept, respectively:

$$\log\frac{B_L}{B_R} = \log b + a\log\frac{R_L}{R_R}.$$

Equation 2.2

The effects on choice of other reinforcer variables, such as magnitude or hedonic quality, can be included by concatenation of additional ratios (multiplicatively in Equation 2.1, additively in Equation 2.2; Baum & Rachlin, 1969; Baum, 1974; Logue, Rodriguez, Peña-Correal & Mauro, 1984). Baum (1979) showed that the generalized matching law provided an excellent quantitative description of choice in concurrent schedules, accounting for more than 90% of the variance in response allocation across a wide range of studies.

A conditioned reinforcer is a stimulus that has acquired the ability to act as a reinforcer through a history of pairing with a biologically-relevant incentive such as food (see Williams, 1994, for review). Traditionally, the terminal-link stimuli in concurrent chains have been viewed as conditioned reinforcers, so that initial-link response allocation provides a measure of their relative effectiveness or value. According to this view, concurrent chains is a concurrent schedule of conditioned reinforcement. This perspective has motivated the development of models for concurrent chains based on the generalized matching law. The assumption has been that initial-link response allocation matches the relative rate of conditioned reinforcement (i.e., rates of terminal-link entry), and that relative terminal-link value combines multiplicatively:

$$\frac{B_L}{B_R} = b\left(\frac{R_L}{R_R}\right)^a\left(\frac{V_L}{V_R}\right)$$

Equation 2.3

where R_L and R_R correspond to rates of conditioned reinforcement, and V_L and V_R to terminal-link value. Equation 2.3 requires that in logarithmic terms, the effects on choice of relative conditioned reinforcement rate and value are additive and independent. Support for this assumption was provided by Berg and Grace (2004), who found that sensitivity to relative conditioned reinforcement rate was constant for different pairs of terminal-link schedules.

Models for concurrent chains based on the generalized matching law include the contextual choice model (Grace, 1994) and hyperbolic value-added model (Mazur, 2001). These models differ in quantitative details, but are similar in that both assume that value is determined as a function of the reinforcer delay signalled by terminal-link onset. Moreover, both models predict that if the initial-link schedules are constant and equal, and the terminal-link delay ratio is varied with the average delay held constant, initial-link response allocation will be a linear function (in logarithmic terms) of the terminal-link delay ratio:

$$\log\frac{B_L}{B_R} = \log b + a\log\frac{1/D_L}{1/D_R},$$

Equation 2.4

where D_L and D_R are the reinforcer delays signalled by the terminal links. These models effectively specify a set of constraints under which an analogue to generalized matching in concurrent schedules (Equation 2.2) should be obtained. Although the slope of the function (i.e., sensitivity to relative delay) can depend on schedule type (e.g., fixed or variable

delays; Grace & Bragason, 2005; Killeen, 1968, MacEwen, 1972) and absolute initial- and terminal-link duration (Fantino, 1969), these models provide a quantitative description of the data comparable to the generalized-matching law for concurrent schedules, typically with 90% or more of the variance in initial-link response allocation explained.

Although initial-link response allocation is the most frequently studied dependent variable in concurrent chains, Staddon and Cerutti (2003) proposed that the time spent waiting in the initial links after reinforcement might be a fundamentally more important measure. They showed that an extension of linear waiting theory (Wynne & Staddon, 1988) was able to predict some well-known results in concurrent chains such as preference for variable-interval over fixed-interval terminal links (Killeen, 1968) and the effect of overall initial-link duration on preference (Fantino, 1969). Cerutti and Staddon (2004) observed a power relationship between initial-link response ratios and relative wait times in concurrent chains, which suggests that both measures may derive from a single process. However, Staddon and Cerutti's (2003) linear-waiting model has not been developed in sufficient detail to allow it to provide a quantitative account of response allocation similar to models based on the matching law (Grace, 1994; Mazur, 2001).

An alternative account, related to cognitive models for timing (Gallistel & Gibbon, 2000; Gibbon, Church, Fairhurst & Kacelnik, 1988), is that choice in concurrent schedules and concurrent chains is based on two different decision processes. In concurrent schedules, the alternatives are available simultaneously and the subject is assumed to allocate their behavior to maximize overall gain. Gallistel and Gibbon (2001) proposed that an 'allocation' mechanism operates to produce matching in concurrent schedules, such that the relative probability of switching from an alternative is determined by relative reward

income (i.e., the product of reinforcement rate and magnitude). By contrast, because the terminal-link schedules are mutually exclusive, subjects in concurrent chains are assumed to make initial-link choices according to an 'opting' mechanism, in which the alternative that has the subjectively greater return is chosen exclusively. According to this view, reinforcer delays are represented in a memory that is repeatedly sampled during the initial links, with subjects responding to the alternative with the shorter remembered delay. A 'winner take all' process similar to a categorical discrimination underlies choice, and, given sufficiently accurate representation and recall of delays in memory, initial-link response allocation should be a nonlinear (i.e., step) function of the delay ratio. However, an 'opting' model can predict that response allocation is a linear function of the delay ratio provided there is sufficient variance in the memorial representations.

Models for timing are able to explain an impressive range of empirical phenomena (Church, 2002). Timing and choice have long been studied separately, so the possibility of a single theoretical framework to integrate these areas is an appealing one. Yet evidence in favor of key assumptions of the cognitive proposal – that a categorical or winner-take-all mechanism underlies choice in concurrent chains, and that choice and timing are mediated by a common representation of reinforcer delay – is lacking. Gallistel and Gibbon (2001) describe an unpublished experiment by Brunner, Gallistel, Fairhurst and Gibbon in which pigeons were trained to respond to VI schedules associated with different stimuli. When the stimuli were later presented concurrently in choice probes, matching was not obtained; instead, subjects responded exclusively to the alternative associated with the richer schedule. Similar results were reported recently by Crowley and Donahoe (2004; Donahoe & Palmer, 1994, pp. 112-113). Gallistel and Gibbon argued that the reason why the pigeons demonstrated exclusive preference for the richer alternative was because they had

learned in the initial training that the schedules could not be concurrently exploited. Consequently, the opting mechanism determined choice in the concurrent probes, leading to exclusive preference. Although these results are consistent with the view that non-exclusive choice requires a history of switching – that is, subjects must learn to match – they do not provide direct evidence that an opting process determines choice in concurrent chains.

Provided pigeons have a history of switching, choice responding in probes between alternatives that have been trained separately can demonstrate matching. Grace and Savastano (2000) exposed pigeons in two different concurrent-chains procedures within sessions, and later arranged choice probes in which terminal-link stimuli were presented concurrently. They found that response allocation in the probes was not exclusive, but matched the ratio of reinforcer delays, as predicted by generalized matching models (Grace, 1994). If the pigeons' initial-link choices had been determined by an opting mechanism, probe preference should have been exclusive, because they had no prior opportunity to learn that the terminal links could be concurrently exploited. Unlike Brunner et al.'s subjects, Grace and Savastano's pigeons did have a history of switching during the initial links, and presumably that was the critical factor that enabled non-exclusive choice during the probes. Nevertheless, because choice in the probes matched the delay ratio, Grace and Savastano's (2000) data are consistent with the view that responding in the initial links is determined by the relative value of the terminal-link stimuli.

The hypothesis that choice and timing should be mediated by a common representation of reinforcer delay has been tested in studies using a modified concurrent-chains procedure introduced by Grace and Nevin (1999). Terminal-link responding is

reinforced according to fixed-interval (FI) schedules, but a percentage of terminal-link presentations (e.g., 25%) extend well past the FI value and end in extinction. These 'no food' presentations are analogous to trials in the peak procedure, which is commonly used in research on timing (Roberts, 1981). As a result, the modified concurrent chains provides convergent operations to measure choice and timing (Garner, Hake & Eriksen, 1956; Grace, 2001).

In Grace and Nevin's (1999) experiment, pigeons received baseline training with FI 40 s and FI 20 s terminal-link schedules. After 25 baseline sessions, subjects demonstrated a strong preference for the initial link leading to the FI 20 s, and the location of peak response rate on no-food trials corresponded closely to the schedule durations. The pigeons then received 25 sessions of training in a multiple peak procedure using the same stimuli as the concurrent chain terminal links. During this phase of training, the initial links were replaced by an intertrial interval, and the FI 40 s schedule was changed to FI 10 s. Location of peak response rate on no-food trials quickly adapted to the FI 10 s. The pigeons were then returned to concurrent chains. All subjects continued to prefer the initial link leading to the FI 20 s, and response allocation changed only gradually across sessions. But the critical result was that subjects continued to time accurately the delays from the multiple peak phase (10 s and 20 s) when returned to concurrent chains. Thus, initial-link response allocation favored the longer terminal link even though subjects 'knew' what the actual delays were, based on their responding during the terminal links. This result appears to challenge the view that initial- and terminal-link responding are mediated by a common representation of reinforcer delay.

The present study is based on recent findings that pigeons' choice in concurrent chains can adjust rapidly when terminal-link delays are changed frequently (Grace, Bragason & McLean, 2003; Grace & McLean, 2006). In these studies, the location of the shorter terminal-link FI schedule remained constant within sessions but was changed randomly across sessions. After 30 to 60 sessions of training, choice was determined by the delay ratio in the current session with little or no effect of prior history.

Our plan was to use Grace and Nevin's (1999) modified concurrent-chains procedure with frequently-changing terminal links, to study the acquisition of choice and timing within sessions. Substantial evidence shows that temporal discrimination by an animal trained to respond to one FI schedule can adapt to the presentation of a new FI schedule within just a few trials in single- (Higa, 1997; Higa, Thaw & Staddon, 1993; Innis, Mitchell & Staddon, 1993) and multiple-schedule procedures (Higa, Wynne & Staddon, 1991; Guilhardi & Church, 2005; Lejeune, Ferrara, Simons & Wearden, 1997).

Our experiment studied two conditions which differed in terms of the number of FI schedule values used for the terminal links. In the minimal-variation condition, a single pair of schedules was used (FI 10 s and FI 20 s), whereas in the maximal-variation condition, schedule values for each session were sampled from a potentially infinite population of values. Throughout the experiment, the location of the shorter terminal link changed pseudorandomly from day to day. Because subjects produced acquisition curves every session, we anticipated that our procedure would yield significantly more data to test for convergence of choice and timing than was available in previous studies. In addition, the large number of delay ratios in the maximal-variation condition was expected to provide

sufficient statistical power to distinguish linear from nonlinear functional relations for choice.

2.2.3 Method

2.2.3.1 Subjects

Five pigeons of mixed breed and sex, numbered 111, 112, 115, 116 and 117, were maintained at 85% ad libitum weight ± 15 g through appropriate post-session feedings of a grain, chickpea and cod liver oil mixture. Pigeons were housed individually in a vivarium with a 12hr:12hr light:dark cycle (lights on at 0730h) with free access to water and grit. All had previous experience with a variety of choice procedures including concurrent chains, but had not previously served in experiments in which terminal-link schedules were frequently changed, or been exposed to timing-related procedures.

2.2.3.2 Apparatus

Five standard three-key operant chambers were used, measuring 32 cm deep x 34 cm wide x 34 cm high. The keys were 21 cm above the floor and arranged in a row 10 cm apart. In each chamber there was a houselight located above the center key and a grain magazine with a 5 x 5.5 cm aperture was centered 6 cm above the floor. The houselight provided general illumination at all times except during reinforcement delivery. The magazine contained wheat and was illuminated during reinforcement. A force of approximately 0.15 N was necessary to operate each key. Each chamber was enclosed in a sound-attenuating box, and ventilation and white noise were provided by an attached fan. Experimental events were controlled through a microcomputer and MEDPC® interface located in an adjacent room.

2.2.3.3 Procedure

Because subjects were experienced, training began immediately in a concurrent chains procedure. With few exceptions, sessions were conducted daily at approximately the same time (1100h).

Sessions ended after 72 initial- and terminal-link cycles had been completed or 70 minutes, whichever came first. At the start of each trial, the side keys were illuminated white to signal the initial links. A terminal-link entry was assigned pseudorandomly to the left or right key, with the constraint that in every block of 12 cycles, 6 entries were assigned to each key. An initial-link response produced a terminal-link entry if: (a) it was made to the pre-selected key; (b) an interval selected from a VI 8-s schedule had timed out; and (c) a 1 s changeover delay was satisfied, i.e., at least 1 s had elapsed after a changeover to the side where terminal-link entry was arranged.

The VI 8-s initial-link schedule did not begin timing until the pigeon first pecked either key. In this way, pausing after completion of the terminal links was excluded from initial-link time. The VI 8-s schedule contained 12 intervals constructed from an arithmetic progression, a, $a + d$, $a + 2d$, ..., in which a equals one-twelfth and d equals one-sixth the schedule value. Intervals were sampled without replacement.

Terminal-link entry was signalled by extinguishing the side keys and lighting the center key either red or green. The color of the center key depended on whether a left or right initial-link response had produced the terminal link (red – left, green – right). Terminal-link responses were reinforced with access to grain according to FI schedules. Of the six terminal-link entries that were scheduled in a block for each alternative, there were five food trials and one no-food trial, determined pseudorandomly. On the food trials,

after the FI schedule value had elapsed, the first center-key response was reinforced. During reinforcement, all keylights and the houselight were extinguished and the grain feeder was raised and lighted for 3 s. A 5-s limited hold was in effect, such that if a response was not made within 5 s after the FI schedule had elapsed, the terminal link ended and no reinforcement was delivered. All subjects responded consistently on food trials such that scheduled reinforcers were rarely lost. On no-food trials, the center key was lighted for 60 s and no reinforcement was delivered. For both types of trials, after a terminal link ended the side keys were lighted white signalling the beginning of the initial links and the next cycle.

There were two conditions in the experiment, which differed in the number of delays that were used for the terminal-link schedules. In the minimal-variation condition, the FI schedule values were always 10 and 20 s. Across sessions, the location of the 10-s terminal-link delay changed according to a pseudorandom binary sequence (PRBS) similar to that used by Hunter and Davison (1985). For each block of 31 sessions, the terminal link associated with the 10-s delay was Left (L), L, L, Right (R), R, L, R, R, R, L, R, L, R, L, L, L, L, R, L, L, R, L, R, R, L, L, R, R, R, R, R.

In the maximal-variation condition, terminal-link delays for each session were sampled from a potentially infinite population of values. The maximal-variation condition was designed to be comparable to the minimal-variation condition in that a) the location of the shorter terminal-link delay was changed across sessions according to the 31-step PRBS; b) the expected log immediacy ratios (i.e., log[1/DelayL / 1/DelayR]) were log(2) and log(1/2), respectively, for sessions in which the left and right alternatives were associated with the shorter delay; and c) the mean terminal-link delay was 15 s. To accomplish this,

the terminal-link schedule values for each session were determined by a pseudorandom number generator subject to the constraints that the log immediacy ratios were uniformly distributed between log(4) and log(1/4) and the two delays summed to 30 s for each session. In this way, the expected log immediacy ratio for the maximal-variation condition was the same as that in the minimal-variation condition and was equally likely to be more or less extreme than log(2) or log(1/2).

Pigeons 111 and 112 received 93 sessions of the minimal-variation condition (3 PRBS presentations), followed by 62 sessions of the maximal-variation condition (2 PRBS presentations), and 62 sessions (2 PRBS presentations) of replication of the minimal-variation condition. Pigeons 115, 116 and 117 received the same number of sessions of training but the order of conditions was reversed (i.e., maximal variation, minimal variation, maximal variation).

Measures of performance on individual no-food trials were obtained using the method of Cheng and Westwood (1993) which assumes that responding conforms to a break-run-break pattern. Responses from individual no-food trials were sorted into 1-s bins. The time of occurrence of the first response of three consecutive filled bins was designated the start time. The time of occurrence of the last response before three consecutive empty bins was designated the stop time. We used an alpha level of .05 for all statistical tests.

2.2.4 Results

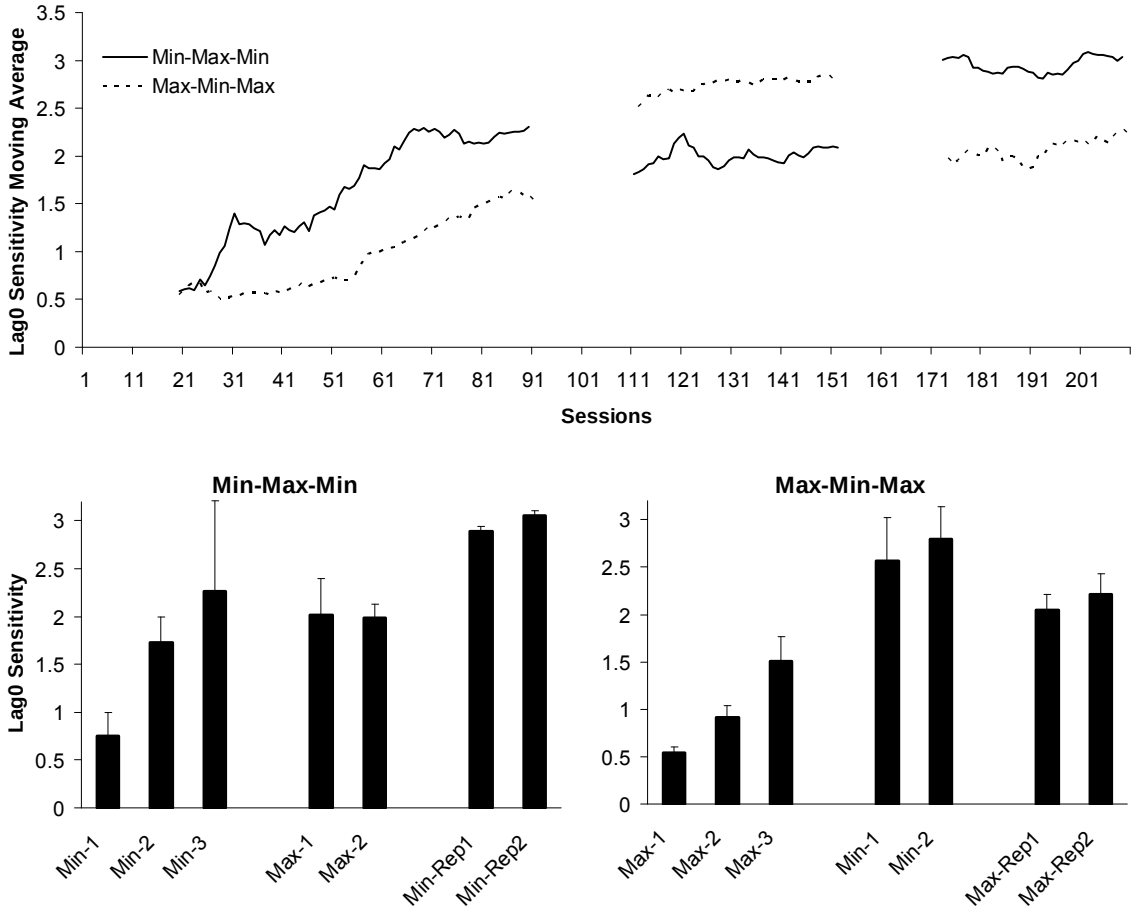

Figure 2.1. Top panel: A moving average of Lag 0 sensitivity to immediacy coefficients, calculated over 20 sessions, for pigeons that first received minimal variation (Min-Max-Min) and maximal variation (Max-Min-Max). Lower panels: Lag 0 sensitivity to immediacy coefficients across successive 25-session blocks comprising the last 75 sessions of Condition 1 and the last 50 sessions each of Conditions 2 and 3. Bars show standard error.

To provide a broad assessment of the acquisition of choice, we calculated sensitivity to immediacy across the entire experiment. For each subject, Equation 2.4 was fitted to the log initial-link response ratios from the second half of the session for successive blocks of 20 sessions. The top panel of Figure 2.1 shows the resulting values of sensitivity to immediacy (i.e., *a* in Equation 2.4), averaged across subjects that were exposed to a minimum-, maximum-, and minimum-variation sequence of conditions (solid line), or the reverse (dashed line). For both sequences, sensitivity increased over the first condition but

was approximately stable during the second and third conditions, and appeared to be greater for the minimal-variation condition. The lower panels of Figure 2.1 show sensitivity values calculated over the last 75 sessions of the first condition, and the last 50 sessions of the second and third conditions, for separate 25-session blocks. Sensitivity increased during the first condition, but was not systematically different between the two blocks of 25 sessions for the second and third conditions. Individual-subject data corresponding to those in the lower panels of Figure 2.1 were entered into a repeated-measures analysis of variance (ANOVA) with order as a between-subjects factor, and condition (minimal- or maximal-variation) and block as within-subjects factors. The main effect of condition was significant, $F(1,3) = 17.89$, $p < .05$, confirming that sensitivity was greater in the minimal-variation condition, but the effects of block and order, and all interactions were not significant. This shows that sensitivity was not significantly different between the two blocks in each condition, and did not depend on order. Accordingly, subsequent analyses are based on data from the last 50 sessions for the second and third conditions.

Figure 2.2 shows response allocation during the initial links in the minimal-variation condition for each of the 6 blocks of 12 cycles within sessions, measured as the logarithm of the ratio of responses to the left and right alternatives. Data are shown separately for sessions in which the terminal-link schedules were FI 10 s FI 20 s and FI 20 s FI 10 s, and were averaged across subjects. For all subjects, initial-link response allocation shifted towards the alternative associated with the shorter terminal link, reaching stability by the third block.

2.2.4.1 Minimal-Variation Condition

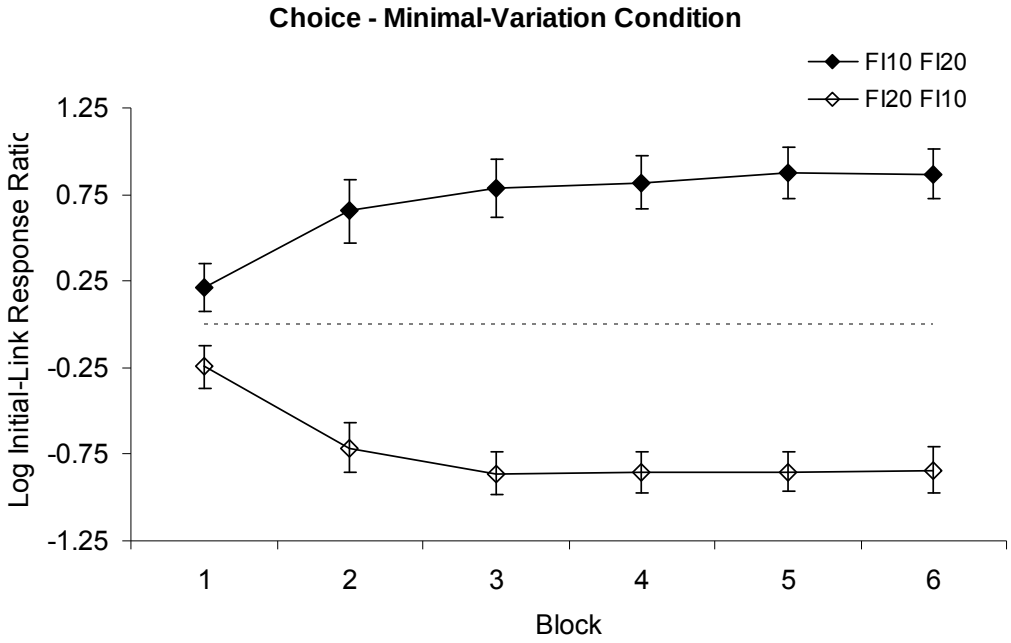

Figure 2.2. Log initial-link response ratios as a function of within-session block in the minimal-variation condition. Filled data points represent the log initial-link response ratio (left/right) from a block of 12 cycles, averaged across subjects and those sessions when the FI 10-s delay was associated with the left alternative. Open data points represent equivalent data for those sessions when the FI 10-s delay was associated with the right alternative. Bars show standard error. The dashed line represents indifference (i.e., an equal number of responses made to the left and right initial links).

The strength of preference for the shorter terminal link may be quantified by the generalized-matching model defined by Equation 2.4. According to Equation 2.4, response allocation is a linear function of the log immediacy ratio, with slope = *a* and intercept = log *b*. Typically, values of *a* obtained with FI terminal links are greater than 1, indicating that response allocation is more extreme than the immediacy ratio (i.e., overmatching; Grace, 1994).

Equation 2.4 was fitted to data from individual subjects for each block. Averaged across subjects, sensitivity values were 0.75 (*SE* = 0.06), 2.26 (*SE* = 0.16), 2.74 (*SE* = 0.21), 2.80 (*SE* = 0.20), 2.91 (*SE* = 0.19) and 2.94 (*SE* = 0.21) for the first through sixth blocks respectively. These values show that pigeons' response allocation strongly favored the

initial link associated with the FI 10-s terminal link. Averaged across the last session half (Figure 2.1), the sensitivity value (2.84) indicates a response ratio of 7.17:1 for the FI 10-s alternative. This value is comparable to the average sensitivity of 2.92 reported by Grace (1994) in his reanalysis of steady-state studies with FI terminal links.

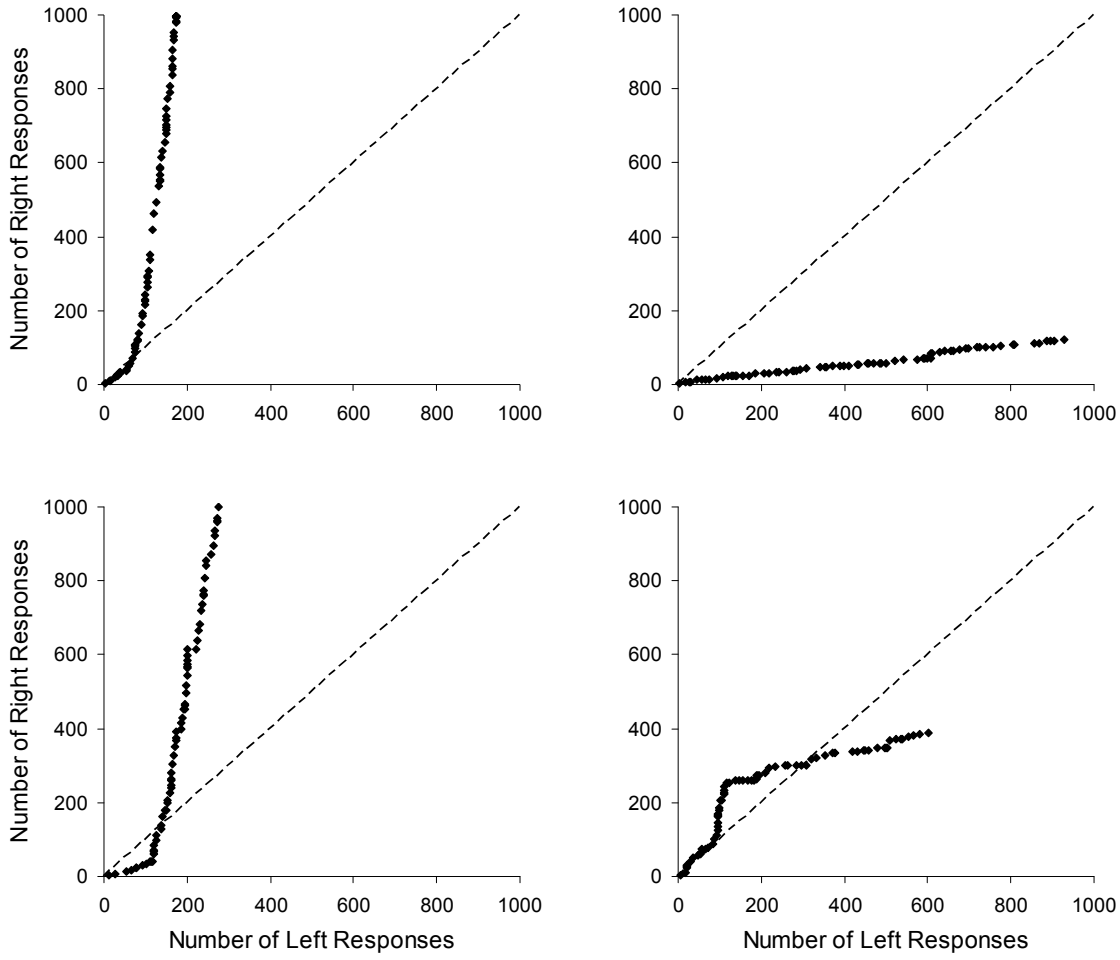

Figure 2.3. Representative scatterplots. Each panel plots the cumulative number of responses made to the right initial link (*y*) against the cumulative number of responses made to the left initial link (*x*). There are 72 points in each panel, with each point representing the cumulative response totals up to and including that trial. The dashed line represents equal responding to the two initial links. Data are shown for four minimal-variation sessions for a representative subject (Pigeon 116).

Figure 2.2 suggests that the strength of response allocation increased gradually. Was this representative of performance in individual sessions? We examined the nature of within-session changes in response allocation by plotting, for each cycle, the cumulative number of responses to the left initial link against the cumulative number of responses to

the right initial link made in the session (Gallistel, Mark, King & Latham, 2001). Scatterplots were produced for all subjects and sessions, and could be described in terms of a few general categories. Figure 2.3 shows representative plots for one subject (Pigeon 116). Each data point represents one cycle. A majority of sessions for all subjects conformed to the pattern of responding depicted in the upper-left panel of Figure 2.3: At first, pigeons made approximately the same number of responses to the left and right keys but at some point switched to prefer one alternative, with the magnitude of this preference remaining invariant for the rest of the session (as evidenced by a constant slope in the scatterplots). Infrequently, other patterns were observed, for example responding that: favored a particular key throughout (upper right panel); favored one key initially and then switched (lower left); or switched multiple times (lower right panel). Inspection of scatterplots revealed that when switches in preference occurred, they were generally abrupt.

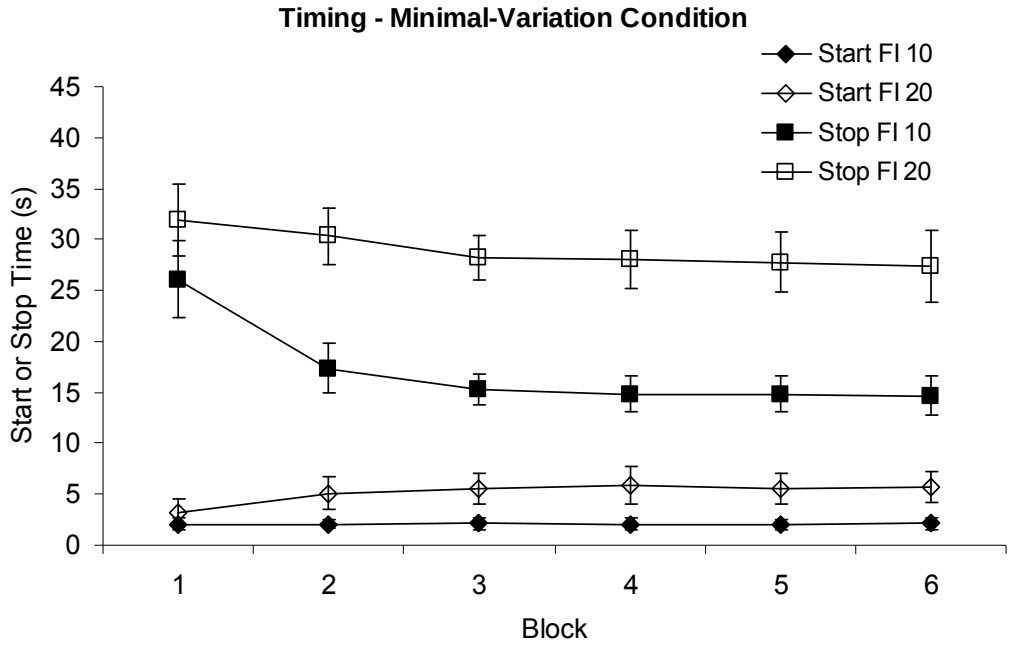

Figure 2.4. Start and stop times as a function of block. Each data point represents the mean start or stop time across subjects from the minimal-variation condition for a particular peak trial and terminal-link schedule. Start times are denoted by diamonds, stop times by squares. Data points are filled for FI 10-s and open for FI 20. Bars represent standard error.

Temporal control of terminal-link responding was measured by start and stop times from individual no-food trials. Figure 2.4 shows mean start and stop times by block (exactly one left and one right no-food trial occurred in each block of 12 cycles), separately for FI 10 and FI 20 terminal links. Individual data from Figure 2.4 were entered into a repeated-measures ANOVA with schedule and block as factors. For stop times, main effects of schedule, $F(1,4) = 410.65$, $p < .001$, and block, $F(5,20) = 54.49$, $p < .001$, were significant, as was their interaction, $F(5,20) = 33.33$, $p < .001$. Similar results were obtained with start times: The main effects of schedule, $F(5,20) = 18.76$, $p < .05$, block, $F(5,20) = 5.99$, $p < .01$, and their interaction, $F(5,20) = 8.01$, $p < .001$, were all significant. Post-hoc tests (Tukey HSD) found that stop times were significantly different for FI 10 and FI 20 for all blocks (although the difference was smaller for the first block), and that for each schedule did not change after the third block. Post-hoc tests also showed that start times were significantly different for FI 10 and FI 20 for all but the first block, and did not change after the second block for FI 20, or at all for FI 10. This suggests that at the beginning of the session, subjects began responding to both terminal links as if they were the FI 10 and ceased responding on no-food trials as if both were the FI 20. However, temporal control developed rapidly and differential responding stabilized prior to the end of the first half of the session.

Table 2.1. A moving average of Lag 0 sensitivity to immediacy coefficients, calculated over 20 sessions, for pigeons that first received minimal variation (Min-Max-Min) and maximal variation (Max-Min-Max). Lower panels: Lag 0 sensitivity to immediacy coefficients across successive 25-session blocks comprising the last 75 sessions of Condition 1 and the last 50 sessions each of Conditions 2 and 3 ("Min-Rep" for Pigeons 111 and 112, "Max-Rep" for Pigeons 115, 116 and 117). Bars show standard error.

	Block					
	1st	2nd	3rd	4th	5th	6th
Start						
Slope	0.11	0.30	0.34	0.38	0.35	0.37
SE	*0.04*	*0.07*	*0.07*	*0.08*	*0.06*	*0.08*
Intercept	1.04	-0.95	-1.28	-1.68	-1.42	-1.57
SE	*0.33*	*0.76*	*0.67*	*0.77*	*0.60*	*0.81*
VAC	0.06	0.28	0.37	0.32	0.39	0.41
SE	*0.02*	*0.06*	*0.08*	*0.06*	*0.08*	*0.10*
Stop						
Slope	0.59	1.29	1.30	1.33	1.32	1.33
SE	*0.08*	*0.07*	*0.07*	*0.06*	*0.07*	*0.07*
Intercept	20.24	4.56	2.40	1.89	1.99	2.05
SE	*1.39*	*0.87*	*0.77*	*0.84*	*0.82*	*0.98*
VAC	0.14	0.64	0.75	0.69	0.73	0.69
SE	*0.03*	*0.06*	*0.04*	*0.04*	*0.05*	*0.06*

The degree of temporal control was quantified in two ways. First, we regressed start and stop times on FI values for each block. Table 2.1 shows the results. For all subjects, regression slopes for both start and stop times increased over the first half of the session, stabilizing by the fourth block. Averaged across the second session half, the slopes for start and stop times were 0.37 (SE = 0.07) and 1.33 (SE = 0.06), respectively. This confirms results in Figure 2.4 that both start times, and, to a greater extent, stop times, increased with FI duration.

Second, we calculated coefficients of variation for start and stop times for individual sessions across the last three peak trials, when behavior had stabilized. The coefficient of variation is the standard deviation divided by the mean, and is widely used in research on timing as an index of relative accuracy (Church, 2002). To reduce bias due to small sample sizes, standard deviations were calculated by dividing by $N - 1$ rather than N.

This results in a more conservative (i.e., larger) estimated standard deviation for each session. For start times, coefficients of variation averaged across subjects were 0.27 (*SE* = 0.04) and 0.34 (*SE* = 0.02) for FI 10 and FI 20, respectively. Corresponding values for stop times were 0.11 (*SE* = 0.00) and 0.10 (*SE* = 0.01). A repeated-measures ANOVA revealed a significant main effect of measure (Start or Stop), $F(1,4) = 93.20$, $p < .001$, but not of schedule, $F(1,4) = 1.32$, *ns*, or the measure x schedule interaction, $F(1,4) = 2.00$, *ns*. The similarity of coefficients of variation for the FI 10 and FI 20 schedules, for both start and stop times, is consistent with the scalar property, which requires variability of timing-related behavior to increase proportionally with the mean (Gibbon, 1977).

2.2.4.2 Maximal-Variation Condition

Terminal-link schedule values were generated pseudorandomly for each session in the maximal-variation condition, which precludes averaging over repeated presentations of a given schedule value as in Figures 2.2 and 2.4. Thus, to examine relationships between response allocation, temporal control, and terminal-link schedule values we first present scatterplots of full-session performance (Figures 2.5 and 2.7). Within-session changes are then summarized by regressions conducted separately for data from each block of 12 cycles. We also examined scatterplots of cumulative responding, similar to Figure 2.3.

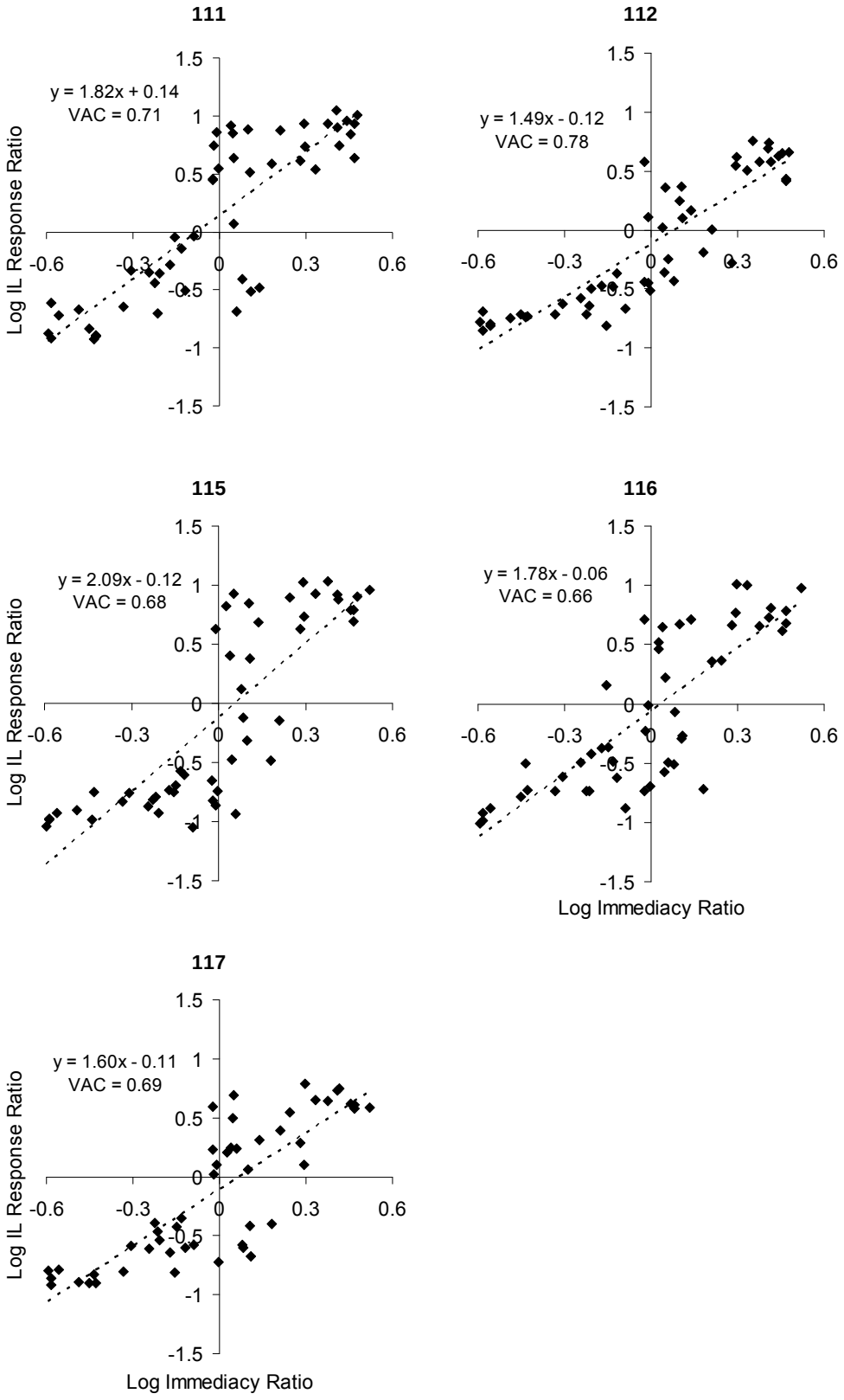

Figure 2.5. Log initial-link response ratios as a function of log immediacy ratios for the maximal-variation condition for all subjects. Each data point represents performance from a single session. Parameters and variance accounted for by linear regression (dashed lines) are also shown.

Figure 2.5 shows response allocation: the log initial-link response ratio as a function of the log terminal-link immediacy ratio for individual subjects. Each data point represents performance from a single session. Dashed lines represent fits of Equation 2.4. As expected, preference for the left alternative increased as a function of the log immediacy ratio, but deviations from linearity were clearly apparent. For all subjects, the data approximately fell into two clusters depending on whether responding favored the left or right alternative. Within each cluster, the sensitivity of response allocation to changes in the log immediacy ratio was less than the sensitivity for the data as a whole (i.e., the regression lines in Figure 2.5). The pattern could also be described as sigmoidal or S-shaped to some extent, with a relatively abrupt shift in response allocation toward the left key as the log immediacy ratio increasingly favored the left key, although all subjects showed a tendency to 'overshoot' near the midpoint (i.e., the immediacy ratio favored the left alternative but the response ratio favored the right, or vice versa).

To confirm the deviations from linearity, we conducted an analysis in which fits of Equation 2.4 were compared with those of a piecewise linear model in which separate linear functions with the same slope but different intercepts were fitted depending on whether response allocation favored the left or right alternative. Specifically, data for each subject in Figure 2.5 were sorted into two groups depending on whether more responses had been made to the left or right alternative and the following model fitted:

$$\text{if} \begin{cases} \log\frac{B_L}{B_R} < 0 : \log\frac{B_L}{B_R} = \log b_1 + a\log\frac{1/D_L}{1/D_R} \\ \log\frac{B_L}{B_R} \geq 0 : \log\frac{B_L}{B_R} = \log b_2 + a\log\frac{1/D_L}{1/D_R} \end{cases}$$

Equation 2.5

If the data in Figure 2.5 were linear, there should be no systematic difference between the fitted values for the intercepts (log b_1, log b_2) in Equation 2.5, and the slopes should not be different from those for the data as a whole. However, for all subjects, log b_1 was always positive (M = 0.44, SE = .07) and log b_2 was always negative (M = -0.48, SE = .04), $t(4) = 6.92$, $p < .01$ and slopes for the linear model (M = 1.76, SE = .10) were always greater than slopes for the piecewise linear model (M = 0.69, SE = .02), $t(4) = 10.25$, $p < .001$. The average variance accounted for (VAC) by the piecewise linear model was 0.93 (SE = .01), compared to 0.70 (SE = .10) for the linear regression. F ratios showed that the incremental variance accounted for by Equation 2.5 was significant for all subjects. This confirms the visual impression from Figure 2.5 that the data tended to fall into two separate clusters depending on whether the left or right alternative was favored, and that sensitivity to immediacy was relatively low within each cluster.

Table 2.2 Results of fitting linear (i.e., the generalized-matching model of Equation 2.4) and piecewise linear (Equation 2.5) models to the data for each block of 12 cycles in the maximal-variation condition. Parameters were obtained for data from individual subjects and then averaged.

	Block					
	1st	2nd	3rd	4th	5th	6th
GML						
Slope	0.67	1.64	1.99	2.13	2.12	2.20
SE	*0.08*	*0.11*	*0.13*	*0.12*	*0.13*	*0.14*
Intercept	-0.02	-0.06	-0.07	-0.07	-0.06	-0.04
SE	*0.06*	*0.06*	*0.05*	*0.05*	*0.06*	*0.06*
VAC	0.35	0.59	0.64	0.68	0.68	0.69
SE	*0.06*	*0.02*	*0.02*	*0.03*	*0.03*	*0.03*
Piecewise Linear						
Slope	0.31	0.60	0.60	0.69	0.67	0.75
SE	*0.06*	*0.04*	*0.01*	*0.05*	*0.05*	*0.11*
Intercept, >0	0.25	0.45	0.58	0.59	0.60	0.61
SE	*0.05*	*0.06*	*0.07*	*0.09*	*0.10*	*0.10*
Intercept, <0	-0.26	-0.49	-0.61	-0.63	-0.63	-0.62
SE	*0.02*	*0.01*	*0.05*	*0.06*	*0.06*	*0.07*
VAC	0.72	0.84	0.91	0.94	0.94	0.95
SE	*0.02*	*0.02*	*0.00*	*0.01*	*0.01*	*0.01*

Next we fitted Equations 2.4 and 2.5 to individual-subject data separately for each block of 12 cycles. Table 2.2 shows the resulting parameter estimates and VAC for both models, averaged across subjects. Slopes for individual subjects were entered into a repeated-measures ANOVA with source (linear or piecewise linear) and block as factors. Main effects of source, $F(1,4) = 83.29$, $p < .001$ and block, $F(5,20) = 47.38$, $p < .001$ were both significant, as was their interaction, $F(5,20) = 32.78$, $p < .001$. Post-hoc tests (Tukey HSD) found that slopes did not change significantly after the third block. This confirms that initial-link response allocation had stabilized prior to the end of the first session half.

The systematic deviation from linearity suggests that the generalized matching model (Equation 2.4) provides an inadequate account of response allocation in the maximal-variation condition. Overall, the pattern of results is consistent with a categorical discrimination in that there was relatively greater sensitivity in response allocation between sessions in which the left or right alternative was favored, than within a group of sessions in which the same alternative was favored. In other words, subjects generally demonstrated a strong preference for the alternative associated with the shorter delay, but by how much the delay was shorter did not appear to have a substantial impact on response allocation.

We compared sensitivity to relative immediacy in the minimal- and maximal-variation conditions based on second-half session performance (Figure 2.1). Averaged across subjects, sensitivity to immediacy in the maximal-variation condition was 2.13 (SE = 0.13), significantly lower than in the minimal-variation condition, 2.84 (SE = 0.22), $t(4) = 4.27$, $p < .05$ Overall, response allocation was more extreme in the minimal-variation condition.

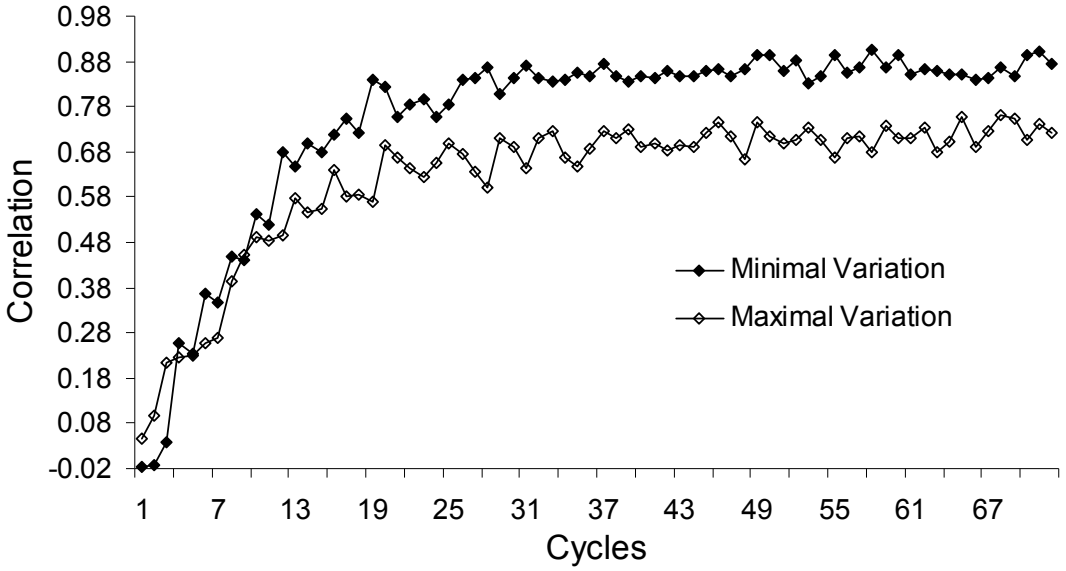

Figure 2.6. Correlations coefficients of log initial-link response ratios with log immediacy ratios as a function of elapsed cycles. Filled points represent data from the minimal-variation condition; open points represent data from the maximal-variation condition.

Acquisition of preference was compared between the conditions in two ways. First, we examined cumulative scatterplots for the maximum-variation condition analogous to those in Figure 2.3. These scatterplots were similar to the representative data in Figure 2.3, and showed that within-session changes in response allocation were generally abrupt in the maximal-variation condition. Second, we calculated correlation coefficients between log initial-link response ratios and log terminal-link immediacy ratios for each cycle. To avoid division by zero, 0.5 was added to each response total. Correlations averaged across subjects are shown in Figure 2.6 for both conditions. This figure is representative of the data for individual subjects and shows that correlations were near zero for the first several cycles, but increased rapidly to stable levels by about the 20th cycle. The average value of the correlation coefficient at stability was greater for the minimum-variation condition, but there was no difference in how quickly stability was reached. A repeated-measures ANOVA found significant effects of condition and cycle, $F(1,4) = 91.73$, $p < .001$ and $F(71,284) = 47.20$, $p < .001$, respectively, as well as their interaction, $F(71,284) = 2.57$, $p <$

.001. Overall, these results concur with the observation based on cumulative scatterplots that pigeons were usually initially indifferent at the start of a session (e.g., upper left panel of Figure 2.3) but that preference developed at an equivalent rate in both conditions. The difference in correlations at stability is consistent with the difference in sensitivity to immediacy between the conditions, and was likely caused by subjects occasional 'misclassifications' – cases in which the relatively longer delay was preferred in the maximum-variation condition. These are evident as data points in the upper left and lower right quadrants of the generalized-matching plots in Figure 2.5.

Figure 2.7 shows start and stop times as a function of terminal-link delay, for individual subjects in the maximal-variation condition. Each data point represents the mean of the start or stop times from all six peak trials for a single terminal link and session. For all subjects, start and stop times increased linearly as a function of delay, although start times tended to become more variable with increasing delay. Averaged across peak trials, terminal links and subjects, the slope and intercept obtained from regressions of stop times on terminal-link delay were 0.98 (*SE* = 0.07) and 10.58 (*SE* = 1.25), respectively, with VAC = 0.62 (*SE* = 0.07). Average slopes and intercepts obtained from regressions of start times on terminal-link delay were 0.31 (*SE* = 0.06) and -0.76 (*SE* = 0.60), respectively, with VAC = 0.55 (*SE* = 0.06).

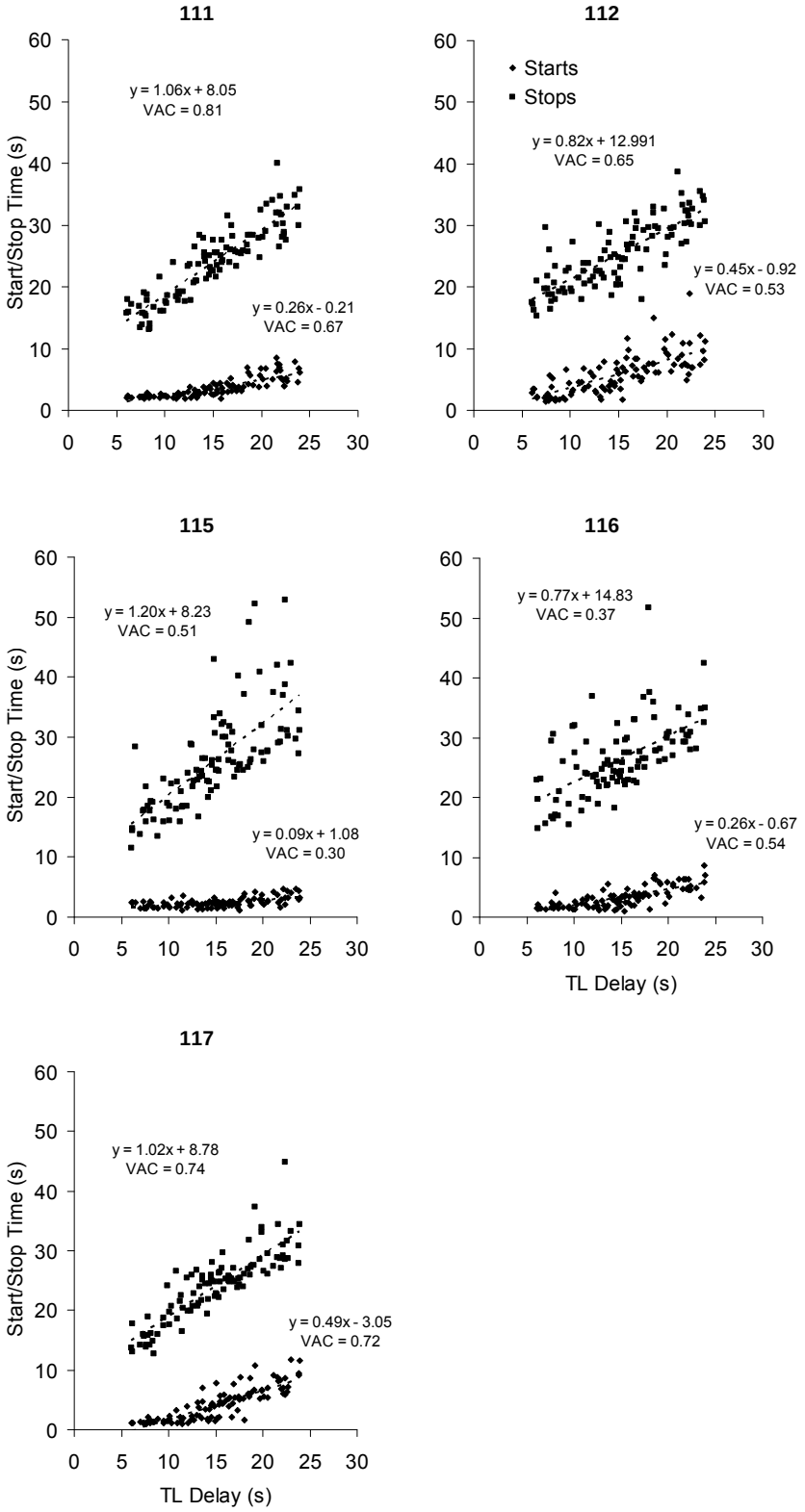

Figure 2.7. Start and stop times as a function of terminal link delay to reinforcement from the maximal-variation condition for all subjects. Each data point represents the mean start or stop time for a single session and terminal-link (left or right) – diamonds for start times and squares for stop times. Parameters and variance accounted for by linear regressions (dashed lines) are also shown.

Table 2.3. Results of regressions of start and stop times on terminal-link delay in the maximal-variation condition for each block of 12 cycles. Slopes and intercepts were obtained from individual data and then averaged across subjects.

	Block					
	1st	2nd	3rd	4th	5th	6th
Start						
Slope	0.13	0.30	0.37	0.34	0.38	0.35
SE	*0.04*	*0.06*	*0.07*	*0.07*	*0.08*	*0.08*
Intercept	1.14	-0.61	-1.29	-1.12	-1.44	-1.18
SE	*0.44*	*0.66*	*0.67*	*0.84*	*0.66*	*0.58*
VAC	0.10	0.28	0.37	0.33	0.38	0.35
SE	*0.04*	*0.05*	*0.04*	*0.07*	*0.07*	*0.05*
Stop						
Slope	0.53	1.00	0.93	1.09	1.09	1.22
SE	*0.09*	*0.07*	*0.09*	*0.08*	*0.10*	*0.09*
Intercept	22.54	10.79	10.39	7.04	7.37	5.31
SE	*1.71*	*0.92*	*1.35*	*1.41*	*1.58*	*1.65*
VAC	0.11	0.42	0.39	0.54	0.51	0.59
SE	*0.03*	*0.05*	*0.07*	*0.08*	*0.09*	*0.07*

Similar to the minimal-variation condition, we regressed start and stop times on FI values for each block. Table 2.3 shows the results. For all subjects, regression slopes for both start and stop times increased over the first half of the session, stabilizing by the fourth block. Averaged across the second session half, the slopes for start and stop times were 0.35 (SE = 0.07) and 1.13 (SE = 0.09), respectively. Individual data from Table 2.3 were entered into a repeated-measures ANOVA with block as a factor. For both start and stop time regression slopes, the effect of block was significant, $F(5,20) = 14.72$, $p < .001$, and $F(5,20) = 10.96$, $p < .001$, respectively. Post-hoc tests (Tukey HSD) found that stop time slopes did not change after the fourth block, and start time slopes did not change after the second block. Because the terminal-link delays for each session were generated pseudorandomly, the stop and start time data demonstrate that acquisition of temporal control was extremely rapid.

To compare temporal control between the minimal- and maximal-variation conditions, we averaged regression slopes over the second session half. For start times, the average regression slopes were 0.37 (*SE* = 0.07) and 0.35 (*SE* = 0.07) for the minimal- and maximal-variation conditions, respectively. Corresponding values for stop times were 1.32 (*SE* = 0.06) and 1.13 (*SE* = 0.09). For both start and stop times, the regression slopes were not significantly different between the conditions, $t(4) = 0.38$, *ns*, and $t(4) = 1.80$, *ns*, respectively. Thus, start and stop times increased similarly as a function of delay in both conditions.

Table 2.4. Slopes and intercepts from linear regressions of start and stop time coefficients of variation on terminal-link delay in the maximal-variation condition. Average start and stop time coefficients of variation (Average CV) and their standard deviation are also shown.

Pigeons	Slope	Intercept	Average CV	(SD)
Start				
111	0.02	0.03	0.28	(0.19)
112	0.01	0.29	0.41	(0.27)
115	0.01	0.08	0.25	(0.16)
116	0.00	0.37	0.37	(0.23)
117	0.01	0.19	0.33	(0.23)
Stop				
111	0.06	0.08	0.09	(0.06)
112	0.00	0.09	0.18	(0.19)
115	-0.05	0.12	0.11	(0.08)
116	0.00	0.12	0.12	(0.10)
117	0.03	0.10	0.11	(0.06)

We calculated coefficients of variation for start and stop times for individual sessions across the last three peak trials, again dividing by *N*-1 to reduce bias when computing standard deviations. As a test of the scalar property, which predicts that the coefficient of variation should be constant as a function of delay, we performed a regression analysis in which coefficients of variation were regressed on terminal-link delay. Table 2.4 shows the results. For all subjects, regression slopes were close to zero, indicating that

coefficients of variation did not vary systematically as a function of delay. Inspection of scatterplots provided further support for this conclusion.

Because coefficients of variation did not change systematically, we calculated averages across delays for individual subjects, as shown in the right-hand columns of Table 2.4. The average coefficient of variation for stop times, 0.12 (SE = 0.01), was significantly lower than the corresponding value for start times, 0.33 (SE = 0.03), $t(4) = 10.83$, $p < .001$. Overall, coefficients of variation in the maximal-variation condition were similar to those obtained in the minimal-variation condition.

2.2.4.3 Analysis of Covariation in Choice and Timing

A major goal of our study was to test whether behaviors related to choice and timing are based on a common representation of delay, through examining patterns of covariation. Because choice is relative measure, our analysis required a relative measure of temporal control. We used the log ratios of start and stop times (i.e., $\log[Start_R / Start_L]$ and $\log[Stop_R / Stop_L]$). These log ratios should be positive or negative when the shorter delay is associated with the left or right terminal link, respectively, and thus be positively correlated with log initial-link response ratios (left/right). However, if choice and timing are determined by a common representation of delay, encoding error should induce additional positive correlation. Specifically, we assume that the delay representation for a particular terminal-link, $DREP_i$, is a function of delay plus encoding error, i.e., $DREP_i = f(D_i) + \varepsilon_i$, where D_i is delay and ε_i is error for terminal link i. If the ratio $DREP_R / DREP_L$ is a determiner of choice and relative timing, then $\varepsilon_R, \varepsilon_L > 0$ should produce additional positive correlation. To test whether this additional correlation was present, we planned to remove variance in log initial-link response ratios and log terminal link start and stop time ratios attributable to $\log(D_R / D_L)$, and examine the residual correlations. Because delay

representations presumably changed during sessions, to reduce the possibility that residual correlations would be attenuated due to averaging we analyzed data at the finest grain available, individual blocks. Thus, choice data consisted of left and right responses aggregated across the 12 cycles in each block; start and stop time ratios were calculated using individual peak trials from the appropriate block.

For all subjects, log initial-link response ratios and log terminal-link start and stop time ratios for individual blocks were computed, and regressed on log terminal-link immediacy ratios. Residuals were then obtained for each regression. We calculated the correlation between residuals for the following regression pairs: a) initial-link response and start time; and b) initial-link response and stop time.

Figure 2.8 illustrates this analysis for two subjects. The upper and lower rows of panels show data for Pigeons 111 and 116, respectively, from the 6th block of the maximal-variation condition. The left and center panels plot log initial-link response ratios and log start time ratios, respectively, as a function of log terminal-link immediacy ratios. Regression lines are included. The right panels show the residual scatterplots – the residuals from the initial-link regression plotted against the residuals from the start time regression for individual sessions. These subjects were selected because they illustrate a non-significant correlation (r = 0.18; Pigeon 111) and a significant positive correlation (r = .52; Pigeon 116).

Figure 2.8 (Next page). Representative scatterplots for two pigeons (111 and 116). Left and center panels show log initial-link response ratio and log start time ratio, respectively, for the final block as a function of log immediacy ratio. Parameters and variance accounted for by linear regressions (solid lines) are also shown. The rightmost panels show residual preference (the deviation of each data point in left panels from its linear regression) as a function of residual start time (the deviation of each data point in center panels from its linear regression). Correlation coefficients (see Table 2.6) are also shown.

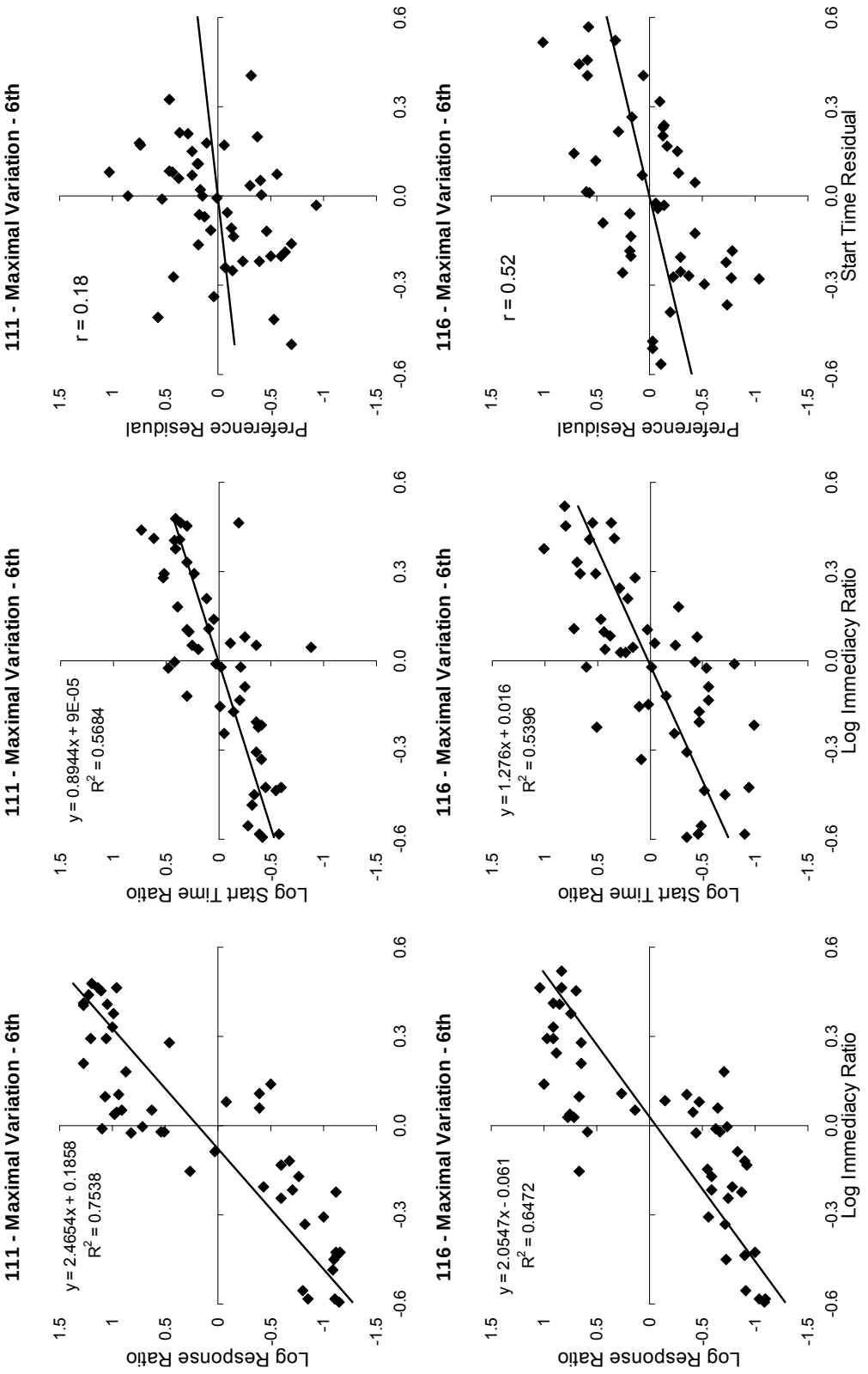
111 - Maximal Variation - 6th
y = 2.4654x + 0.1858
R² = 0.7538
Log Response Ratio
Log Immediacy Ratio
111 - Maximal Variation - 6th
y = 0.8944x + 9E-05
R² = 0.5684
Log Start Time Ratio
Log Immediacy Ratio
111 - Maximal Variation - 6th
r = 0.18
Preference Residual
Start Time Residual
116 - Maximal Variation - 6th
y = 2.0547x - 0.061
R² = 0.6472
Log Response Ratio
Log Immediacy Ratio
116 - Maximal Variation - 6th
y = 1.276x + 0.016
R² = 0.5396
Log Start Time Ratio
Log Immediacy Ratio
116 - Maximal Variation - 6th
r = 0.52
Preference Residual
Start Time Residual

Table 2.5. Analysis of residual correlations, minimal-variation condition. For start time analyses, correlations were computed between residuals from regressions of log initial-link response ratios and log start time ratios on log terminal-link immediacy ratios; for stop time analyses, correlations were computed between residuals from regressions of log initial-link response ratios and log stop time ratios on log terminal-link immediacy ratios. Correlations are shown for individual subjects for each block as listed, and also for pooled data.
Note: *$p < .05$, **$p < .01$, ***$p < .001$.

	Start Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
111	0.25	-0.02	-0.03	0.10	0.07	-0.15
112	0.03	-0.06	0.14	0.10	0.18	0.14
115	-0.15	-0.05	-0.01	0.05	0.11	-0.01
116	0.04	0.29*	0.01	0.09	0.23	0.51**
117	0.46**	-0.21	0.04	0.45**	0.41**	0.48**
Pooled	0.14*	0.00	0.03	.23***	.24***	.26***

	Stop Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
111	0.12	0.13	0.23	0.28*	0.26	0.34*
112	0.23	0.29*	-0.05	-0.04	-0.15	-0.32*
115	0.31*	0.33*	-0.15	-0.13	0.11	-0.07
116	0.23	0.23	0.41**	0.03	0.03	0.15
117	0.21	0.22	-0.07	0.60***	0.44**	0.55***
Pooled	.26***	.31***	0.14*	.23***	.26***	.20**

Residual correlations for all subjects, blocks, and conditions are listed in Tables 2.5 (minimal-variation) and 2.6 (maximal-variation). Although the correlations for individual subjects were variable, the correlations for data pooled across subjects were significantly positive for the minimal-variation condition in 10 out of 12 cases, and for the maximal-variation condition in 12 out of 12 cases. Significant positive correlations were obtained for all subjects in both conditions. In the minimal-variation condition, 16 (out of 60) individual correlations were significant, and 15 of these were positive. There were 26 (out of 60) significant individual correlations in the maximal-variation condition, and all

were positive. Overall, the largest correlations were obtained in the maximal-variation condition and with start times.

Table 2.6. Analysis of residual correlations, maximal-variation condition. For start time analyses, correlations were computed between residuals from regressions of log initial-link response ratios and log start time ratios on log terminal-link immediacy ratios; for stop time analyses, correlations were computed between residuals from regressions of log initial-link response ratios and log stop time ratios on log terminal-link immediacy ratios. Correlations are shown for individual subjects for each block as listed, and also for pooled data.
Note: *$p < .05$, **$p < .01$, ***$p < .001$.

	Start Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
111	0.09	0.33*	0.37**	0.45**	0.18	0.18
112	0.00	0.16	0.20	0.25	0.20	0.38**
115	0.27	-0.07	-0.10	0.13	0.00	-0.03
116	0.24	0.47**	0.62***	0.62***	0.76***	0.52***
117	0.27	0.65***	0.62***	0.79***	0.58***	0.46**
Pooled	0.16*	0.31***	0.36***	0.44***	0.33***	0.29***

	Stop Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
111	-0.13	0.40**	0.45**	0.42**	0.23	0.28*
112	0.18	0.10	0.22	0.02	0.14	0.25
115	0.40**	0.28*	0.17	0.18	0.33*	0.26
116	0.22	0.20	0.30*	0.11	0.17	0.21
117	0.33*	0.26	0.35*	0.39**	0.12	0.38**
Pooled	0.17**	0.21**	0.26***	0.19**	0.19**	0.24***

The nonlinearity of log initial-link response ratios as a function of log immediacy ratios might explain why the magnitude of residual correlations was greater in the maximal-variation condition. Some of the residual variance in choice in the maximal-variation condition was due to this nonlinearity; overall, residual choice variance was greater in the maximal-variation condition in 27 out of 30 cases. Increased residual variance could result in stronger correlations, but cannot explain why the correlations were positive. This is because the corresponding relations for log start and stop time ratios were linear. Therefore the positive residual correlations in the maximal-variation condition were not an artefact of

fitting linear models to two sets of similarly nonlinear data. The question of linearity is irrelevant to the minimal-variation condition, which had only two log delay ratios.

Overall, the positive residual correlations provide some evidence that choice and timing in concurrent chains are mediated by a common representation of reinforcer delay. In the absence of a common representation, initial- and terminal-link responding should be controlled independently by delay. The positive correlation between residual measures of relative initial- and terminal-link responding, with the variance attributable to the delay ratio removed, suggests that these measures were not independent.

2.2.5 Discussion

Choice and timing have usually been studied in isolation, by researchers with different theoretical orientations. The most notable attempt at integration – the application of scalar expectancy theory and rate estimation theory to concurrent chains and related choice procedures by Gibbon, Gallistel and their colleagues (Gallistel & Gibbon, 2000; Gibbon et al., 1988) – rests on assumptions that are largely untested. Here we attempted to lay the foundation for an empirical integration by investigating the acquisition of choice and timing within sessions. We used a concurrent-chains procedure in which some terminal links were 'no food' trials analogous to the peak procedure. In the initial links, pigeons chose between terminal-link stimuli that were associated with different FI schedules, and measures of temporal control were obtained from responding on no-food terminal links. Thus, the procedure provided convergent measurement of choice and timing.

The position of the shorter terminal link varied randomly from day to day according to a pseudorandom binary series. Based on previous research with this procedure (Grace et al., 2003; Grace & McLean, 2006), we anticipated that pigeons' initial-link

response allocation would adapt rapidly to the reinforcer delays (FI schedules) presented in each session. Two conditions were studied, in which the FI schedules were 10 s and 20 s ('minimal variation'), or sampled from a potentially infinite population of values ('maximal variation').

Our primary goals were to test whether initial-link choice was determined by an opting or categorical decision process, as proposed by Gallistel and Gibbon (2000), and to search for evidence of mediation by a common representation of delay. Acquisition of choice was rapid, with response allocation stabilizing in both conditions by about midway through the sessions (Figure 2.2 and Table 2.1). For all subjects, response allocation in the maximal-variation condition was a nonlinear function of the log terminal-link immediacy ratio, with the data falling into two clusters depending on whether the left or right alternative was favored, consistent with a categorical discrimination (Figure 2.5). This provides evidence against the assumption of models based on the matching law that responding in the initial links reflects the relative strength of the terminal-link stimuli as conditioned reinforcers (Grace, 1994; Mazur, 2001). According to these models, response allocation in the maximal-variation condition should have been a linear function of the log immediacy ratio.

Temporal control of responding on no-food terminal links also developed rapidly. For the minimal-variation condition, differential stop times were obtained for the FI 10 s and FI 20 s schedules for the very first no-food trial (first block; Figure 2.3). Because there would have been only three reinforced trials, on average, for each terminal link prior to the first no-food trial, this demonstrates that temporal control emerged extremely rapidly. Similar results were obtained in the maximal-variation condition: The slope for stop time

regressed on terminal-link delay for the first block was significantly positive ($M = 0.53$, $t(4) = 5.54$, $p < .01$).

These results provide additional evidence that temporal control can develop rapidly in some situations. Staddon, Higa and colleagues have shown that *wait time*, the latency to first response, is proportional to the interfood interval and can adapt to new intervals within one interfood interval presentation (Higa, 1997; Higa, Wynne & Staddon, 1991; Wynne & Staddon, 1988). Other measures of timing, including peak time (Lejeune et al., 1997) and response rate (Guilhardi & Church, 2005) have been shown to adjust to interval changes within a session. Our study is the first to show that pigeons can time accurately more than one rapidly-changing interval.

In minimal- and maximal-variation conditions, stop and start times for both delays stabilized midway through the session, and values averaged over the second session half were linearly related to terminal-link delay. The break-run-break pattern observed was consistent with reports of responding on individual peak trials of the peak procedure (Cheng & Westwood, 1993; Church, Meck & Gibbon, 1994). As with previous research, the relationships between start and stop times and delay were linear (Church, Meck & Gibbon, 1994; Crystal, Church & Broadbent, 1997). Overall, stop times showed greater temporal control than start times, with less variability and slopes near 1 for all subjects when maximal-variation stop times were regressed on terminal-link delays, consistent with steady-state literature (Church, Meck & Gibbon, 1994). Whereas slopes for stop times as a function of time to reinforcement were comparable, slopes for start times in this experiment were lower than those reported in steady-state peak-interval procedures (Leak & Gibbon,

1995; Crystal, Church & Broadbent, 1997), possibly due to an excitatory effect of the signalled change from initial to terminal link.

One of the more surprising aspects of our data is that the coefficients of variation were lower than those reported previously, particularly for stop times. In steady-state peak procedures, Church, Meck and Gibbon (1994) reported average coefficients of variation of 0.50 and 0.21 for start and stop times, respectively, for intervals ranging from 15 s to 60 s with rats, and Cheng, Westwood, and Crystal (1993) obtained similar values with pigeons (0.30 for start times and 0.25 for stop times) for intervals between 5 s and 25 s. In the present study, average coefficients of variation for start and stop times, respectively, were 0.30 and 0.11 in the minimal-variation condition, and 0.33 and 0.12 for the maximal-variation condition. Both Church, Meck and Gibbon (1994) and Cheng, Westwood, and Crystal (1993) used steady-state designs in which subjects had many sessions of exposure to each delay. Our results, especially for stop times, are counterintuitive and raise the question of how the pigeons were able to learn to time so accurately as well as quickly in the maximal-variation condition. Another striking finding was the failure to find any difference in either acquisition rate or accuracy of timing in our two conditions. It is reasonable to expect that there should have been some savings in the minimal-variation condition due to the repeated training with the same pair of delays, yet none was found. This suggests that the mechanism for rapid temporal learning is memoryless. Given the degree of accuracy our subjects achieved, approaching each session *de novo*, as the pigeons apparently did, would seem highly adaptive (cf. Riccio, Rabinowitz & Axelrod, 1994).

The nonlinear relationship for response allocation as a function of the immediacy ratio contrasts with the linear relationship between measures of temporal control and delay.

Apparently, subjects have access to a continuous representation of time, on which decisions to start and stop responding on no-food trials are based. Decisions about which initial link to prefer, however, are all-or-none, consistent with the opting mechanism proposed by Gallistel and Gibbon (2000).

To test for mediation by a common representation of delay, we examined the covariation of choice and timing within sessions. Our reasoning was as follows: If responding during the initial- and terminal links was based on a common delay representation, then any error or perturbation in that representation should produce additional covariation beyond that expected because measures of choice and timing increased as a function of the delay ratio. Thus, we removed variance in relative measures of choice and timing that could be predicted by the delay ratio, and assessed the correlation between the residuals. The analysis was conducted at the finest grain possible – single pairs of no-food trials and initial-link responding from individual blocks[1]. Error in the representation (in terms of either encoding or retrieval processes) would be predicted to result in a positive correlation between residual measures of relative choice and timing. This prediction was confirmed. Pooled across subjects, correlations were significantly positive in 10 out of 12 cases for the minimal-variation condition and 12 out of 12 cases for the maximal-variation condition (see Tables 2.5 and 2.6). Of the correlations for individual subjects that reached significance, 97% (30 out of 31) were positive. Thus, relative measures of choice and timing shared variance that was not attributable to the delay ratio. This challenges recent studies which have asserted that deciding when to respond and

[1] Although in principle an even finer grain could be obtained from including initial-link responses only from those trials in which a no-food terminal link occurred, this produces a biased measure of preference because some responses necessarily must occur to the alternative associated with the particular terminal link. In any case, correlations calculated using the single-trial initial-link data were similar to those based on the data pooled within blocks reported in Tables 5 and 6.

deciding what response to make are independent processes (Jozefoweiz, Cerutti & Staddon, 2005), and supports the hypothesis that choice and timing in concurrent chains are mediated, at least in part, by a common representation of delay. This representation is likely a continuous magnitude with scalar variability (Gallistel & Gelman, 2000) that is largely, but not entirely, a function of delay.

The positive evidence for common representation obtained in the present study contrasts with previous research that has used the same general procedure in which a peak procedure is embedded within concurrent chains. Using a design similar to reinforcer revaluation studies (e.g., Adams & Dickinson, 1981), Grace and Nevin (1999) found that pigeons given training with a new pair of delays in a multiple peak procedure continued to time those delays accurately when returned to concurrent chains, even though response allocation in the initial links favored the longer delay. Grace, Berg and Kyonka (2006) conducted a series of exploratory multiple regression analyses in which various local measures of initial-link performance (e.g., pause time, visit duration) were used to predict start and stop times on individual no-food trials. They reasoned that mediation by a common representation of delay should produce covariation of measures of responding on individual trials, yet they were unable to find a combination of initial-link measures that could predict a substantial amount of variance in start and stop times. The failure of these previous studies to find clear evidence for mediation is striking, and an obvious question is why the present results were different.

The most salient difference is that our experiment used a procedure in which acquisition curves for both choice and timing were obtained within sessions, whereas previous research used steady-state procedures in which subjects were given a substantial

number of sessions with unchanged contingencies. How this would explain the difference in results is unclear, although it may be that there are other, unspecified sources of variance in responding in steady-state procedures that obscure the covariation expected from common mediation. This question might be addressed by comparing patterns of variance in steady-state and rapid-acquisition procedures.

Overall, the present results provide support for two key assumptions of the cognitive model proposed by Gallistel and Gibbon (2000). We found clear evidence that choice responding is determined by a 'winner take all' or categorical decision process, and the positive residual covariation suggests that choice and timing are mediated, at least in part, by a common representation of delay. An unexpected result was the temporal learning shown by the pigeons in the maximal-variation condition, which was as fast and accurate as that in the minimal-variation condition. Overall, our data show that pigeons' choice and timing behavior can adjust very rapidly when reinforcer delays are changed frequently.

3 Chapter 3 Rapid acquisition of choice and timing: Effects of initial-link duration

3.1 Notes on Experiment 2

Chapter 3 presents Experiment 2, which I published with my supervisor Randolph Grace as "Effects of unpredictable changes in initial-link duration on choice and timing," in the special issue of *Behavioural Processes, 79*. The 2009 issue in which it appeared was dedicated to the proceedings of the 2008 meeting of the Society for Quantitative Analyses of Behavior. Citations of Kyonka and Grace (2009) in Chapter 4 refer to this article. Our goals were to replicate results from the minimal-variation condition of Experiment 1, and to determine how unpredictable changes in initial-link duration across sessions affected initial- or terminal-link performance. Christensen and Grace (2008) presented results of a rapid acquisition concurrent chains experiment in which initial-link duration increased and decreased across successive sessions. They reported a bitonic initial-link effect and described an extension to Grace and McLean's (2006) decision model that could account for it. Experiment 2 differs procedurally from Christensen and Grace's experiment in two ways. First, it included no-food terminal links to enable collection of start and stop times. Second, although the range of possible scheduled initial-link durations was similar, across sessions, the sequence of values was pseudorandom. A critical question was how initial-link performance in Experiment 2 would compare to initial-link performance in Christensen and Grace's experiment. Application of a decision model to this data is presented later, in Chapter 6.

3.2 Experiment 2: Effects of unpredictable changes in initial-link duration on choice and timing

3.2.1 Abstract

Four pigeons responded in a concurrent-chains procedure in which terminal-link schedules were fixed-interval (FI) 10 s and FI 20 s. Across sessions, the location of the shorter terminal-link changed according to a pseudorandom binary sequence. Each session, the variable-interval initial-link schedule value was sampled from a uniform distribution that ranged from 0.01 to 30 s. On some terminal links, food was withheld to obtain measures of temporal control. Terminal-link delays determined choice (log initial-link response ratios) and timing (start and stop times on no-food trials) measures, which stabilized within the 1st half of each session. Preference for the shorter terminal-link delay was a monotonically-decreasing function of initial-link duration. There was no evidence of control by initial-link durations from previous sessions.

Keywords: concurrent chains, initial link effect, rapid acquisition procedure, temporal control, pigeons

3.2.2 Introduction

In typical concurrent-chains procedures (Herrnstein, 1964), subjects respond to two concurrently-presented signaled options or 'initial links' that operate on variable interval (VI) schedules of reinforcement. Responding in initial links produces either of two mutually-exclusive outcomes or 'terminal links' that end with food reinforcement after another delay has elapsed. Most concurrent chains studies have used steady state designs in which the same schedules operate for many sessions. The usual result is that subjects respond more to the initial link preceding the terminal link associated with the relatively shorter delay to reinforcement (see Mazur, 2001, for review).

Preference in steady state concurrent chains, measured as the logarithm of the initial link response ratio once behavior ceases changing systematically, can be affected by other temporal factors, including average duration of initial links. Longer absolute initial-link durations attenuate preference (Fantino, 1969; Mazur 2005), a phenomenon known as the 'initial-link effect.' According to delay-reduction theory (Fantino 1969), the conditioned reinforcement value of terminal links is determined by the reduction in delay to primary reinforcement, relative to the overall average time between initial-link onset and reinforcer delivery, signaled by onset of a terminal link.

To determine whether initial-link duration affects choice in transition, Berg and Grace (2006) investigated effects of relatively long (VI 24 s) and short (VI 8 s) initial-link schedules on preference and temporal acquisition using a successive-reversals design in which the location of the shorter of two terminal link schedule values switched every 20 sessions. They replicated the initial-link effect of steady-state experiments: Response allocation was closer to indifference when the initial-link schedule was long. The

magnitude of change in response allocation over the first three post-reversal sessions was greater when initial-link duration was short before the reversal and long after it, and smaller when initial-link duration was long before the reversal and short after it. Berg and Grace (2006) contended their results were consistent with the theoretical assumptions of delay reduction theory, since response allocation should be more resistant to change if the conditioned reinforcement value of both terminal links was greater when initial links were long (even though preference itself was less extreme).

Recent studies have shown that subjects' response allocation adapts rapidly when terminal-link schedules change unpredictably across sessions (Grace et al., 2003, Grace and McLean, 2006, Kyonka and Grace, 2007). In one condition of Kyonka and Grace's (2007) experiment, terminal-link schedule values were always FI 10 and FI 20 s, but whether a peck to the left or right key produced the shorter terminal-link delay varied across sessions according to a pseudorandom binary sequence. Initial links always operated according to a VI 10 s schedule. One sixth of terminal links were "no food" trials that ended after 60 s without reinforcement. Responding in no food trials provided measures of temporal control (cf. Cheng and Westwood, 1993). Measures of choice and temporal control showed additional covariance beyond that attributable to terminal-link immediacy ratios, and performance stabilized approximately halfway through sessions with no evidence of influence of prior sessions. Similar research using rapid acquisition procedures has shown that response allocation can adapt to unpredictable changes in relative reinforcer rate (Schofield and Davison 1997), magnitude (Maguire et al., 2007) and simultaneous manipulations of multiple reinforcer dimensions (Kyonka and Grace, 2008; Kyonka, 2008).

Christensen and Grace (2008) investigated effects of initial-link duration on preference when terminal-link schedules changed unpredictably each session. Pigeons were exposed to a rapid acquisition concurrent chains procedure in which initial-link schedule value varied systematically between VI 0.01 s and VI 30 s across sessions according to an ascending-descending sequence. Response allocation adjusted to unpredictable changes in immediacy ratio and stabilized within the first half of sessions with no effect of previous sessions' immediacy ratios. Christensen and Grace noted several effects of changing initial-link duration across sessions. They observed an initial-link effect. However, it was bitonic: log response ratios became more extreme as programmed initial-link duration increased from 0.01 to 7.5 s, then decreased as initial link duration increased to 30 s. They showed that an extension of Grace and McLean's (2006) decision model predicted the bitonic effect. They also found that preference was more extreme in the ascending than the descending part of the sequence. Christensen and Grace attributed the difference to "hysteresis," suggesting that preference established in previous sessions was carried over and influenced responding at the start of a new session.

In the present experiment, both initial- and terminal-link schedule values changed pseudorandomly each session. Our primary goal was to determine what effects, if any, initial-link duration has on preference for the shorter delay to reinforcement and on measures of temporal control. In particular, we wanted to determine whether the effect of initial-link duration on response allocation would be bitonic and whether there would be any evidence of hysteresis. We were also interested in comparing the degree of control by log immediacy ratio in this experiment to those in which initial-link duration did not change.

3.2.3 Method

3.2.3.1 Subjects

Four pigeons of mixed breed and sex, numbered 111-114, were maintained at 85% ad libitum weight plus or minus 15 g through appropriate post-session feedings. Pigeons were housed individually in a vivarium with free access to water and grit, and a 12-hr:12-hr light:dark cycle plus windows providing natural light. All had experience with concurrent-chains procedures in which terminal-link delays changed unpredictably across sessions.

3.2.3.2 Apparatus

Four operant chambers (32 cm deep x 34 cm wide x 34 cm high) were enclosed in sound-attenuating boxes containing ventilation fans. Each chamber contained three keys 21 cm above the floor arranged in a row 10 cm apart, a houselight located above the center key, and a grain magazine with a 5 x 5.5 cm aperture that was centered 6 cm above the floor. The houselight provided general illumination at all times except during reinforcer delivery. The magazine, which was illuminated during reinforcement, contained wheat. A force of approximately 0.15 N was necessary to operate each key. Experimental events were controlled through a microcomputer and MED-PC® interface located in an adjacent room.

3.2.3.3 Procedure

Because all pigeons had previous experience, training began immediately in the concurrent-chains procedure. Prior to this experiment, pigeons had responded in a procedure similar to the present one in all respects except that the initial-link schedule was VI 10 s (Kyonka & Grace, 2007). Sessions ended after 72 initial- and terminal-link cycles or 70 minutes, whichever came first.

At the start of a cycle, side keys were lighted white to signal initial links. A terminal-link entry was assigned pseudorandomly to the left or right key with the constraint that in every block of 12 cycles, 6 were assigned to each key. If an interval selected from the initial link schedule had timed out and it satisfied a 1-s changeover delay, a response to the preselected key produced a terminal link entry.

The initial-link schedule did not begin timing until the pigeon first pecked either key. In this way, pausing after the completion of terminal links was excluded from initial-link time. The initial-link schedule contained 12 intervals sampled without replacement and constructed from an exponential progression (Fleshler & Hoffman, 1962). For each session, the initial-link schedule value was determined by sampling from a uniform distribution that ranged from 0.01 to 30 s.

Terminal-link entry was signaled by extinguishing the side keys and lighting the center key. The color of the center key depended on whether a left or right initial-link response had produced the terminal link (red – left, green – right). Terminal-link responses were reinforced with 3 s access to grain according to FI schedules, always FI 10 s and FI 20 s. The (left or right) location of each terminal-link delay varied across sessions according to a 31-step pseudorandom binary sequence similar to the one used by Hunter and Davison (1985), but stayed at a key location within a session. Of the six terminal-link entries that were scheduled in a block for each alternative, five food trials and one no-food trial were determined pseudorandomly. On food trials, after the scheduled interval elapsed, the first center-key response was reinforced. A 5-s limited hold was in effect, such that if a response was not made within 5 s after the FI schedule had elapsed, the terminal link ended and no reinforcement was delivered. On no-food trials, the center key was lighted for 60 s

and no reinforcement was delivered. For both types of trials, after a terminal link ended the side keys were lighted white signaling the initial links and the beginning of the next cycle.

Measures of temporal control on individual no-food trials were obtained using the method of Cheng and Westwood (1993). Responses from individual no-food trials were sorted into 1-s bins. The time of occurrence of the first response from the first instance of three consecutive filled bins was designated the start time. The time of occurrence of the last response before three consecutive empty bins was designated the stop time.

Pigeons 111-113 received 160 and Pigeon 114 received 125 total sessions of training. Data included in these analyses are from the last 50 sessions of training for Pigeons 111, 112 and 114. However, after the first 50 sessions, responding for Pigeon 113 became erratic and thereafter it failed to complete sessions consistently. Thus, we analyzed data from the first 50 sessions for Pigeon 113. For all subjects, 10-session moving-average sensitivity to current-session immediacy ratios did not change systematically over the 50 included sessions.

3.2.4 Results

3.2.4.1 Obtained initial-link durations

Figure 3.1 shows average overall time spent responding in the initial links for each subject and session, plotted as a function of programmed initial-link duration. Obtained initial-link duration was an increasing linear function of the programmed value. For three subjects, best-fitting regression lines accounted for over 87% of the variance. Obtained durations were more variable for Pigeon 113 than for the other subjects. For subsequent analyses reported below, all statistical tests were performed with both obtained and

programmed durations. Because outcomes were similar in all cases, we report only results based on programmed values.

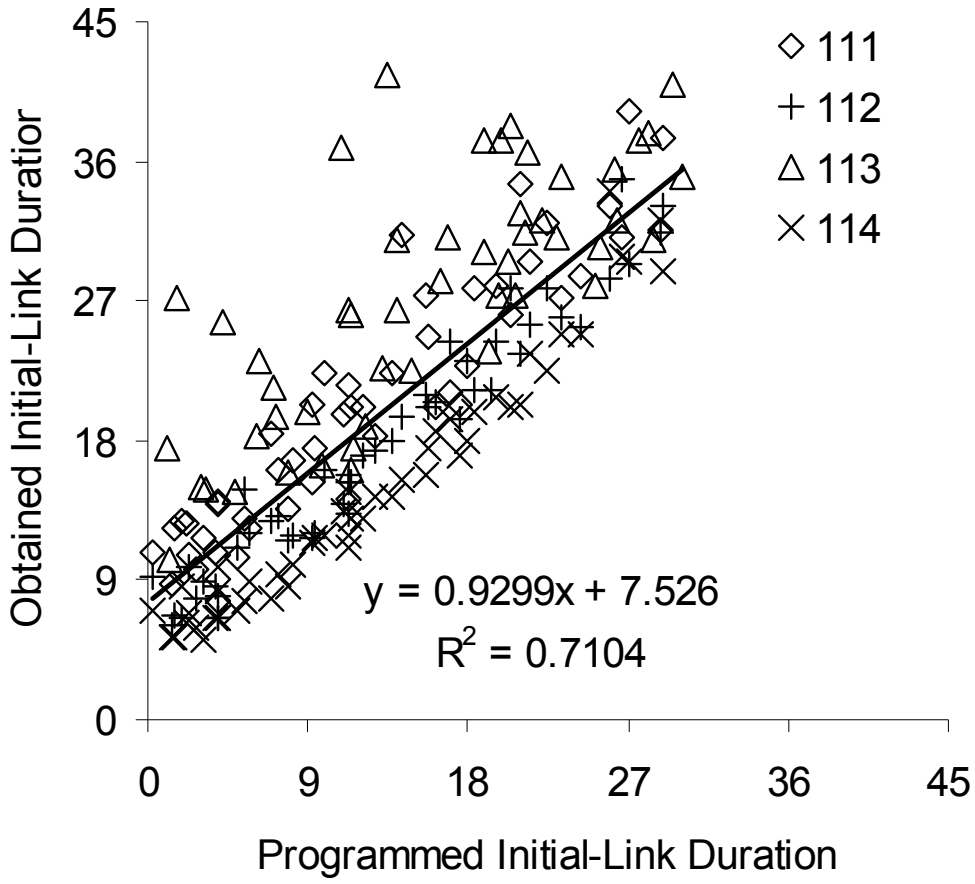

Figure 3.1. Obtained initial-link duration as a function of the programmed value. Diamonds, pluses, triangles and crosses represent initial-link durations from Pigeons 111, 112, 113 and 114, respectively. Parameters and variance accounted for (R^2) by a linear regression (solid line) on data pooled across subjects are also shown.

3.2.4.2 Acquisition of control by terminal-link delay

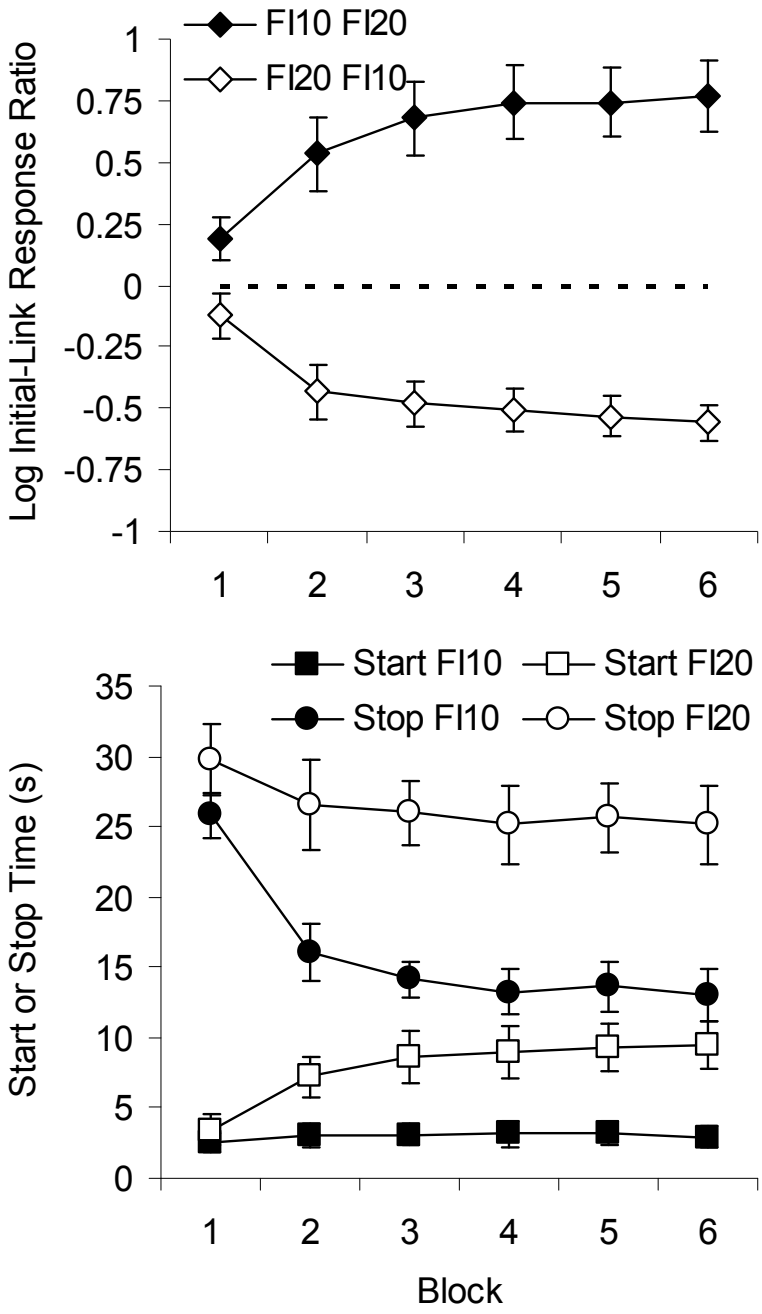

Figure 3.2. The top panel shows log initial-link response ratios as a function of block. Filled data points represent the left/right log initial-link response ratio from blocks of 12 cycles for those sessions in which the FI 10 s delay was associated with the left alternative. Open data points represent equivalent data for sessions in which FI 10 s was associated with the right. Bars show standard error. The dashed line represents indifference – an equal number of responses made to left and right initial links. • The bottom panel shows start times (denoted by square data points) and stop times (circles) from no-food trials of FI 10 s (filled data points) and FI 20 s (open) terminal links as a function of block. Each data point represents the group mean start or stop time for a particular block and terminal-link schedule. Bars represent standard error.

We examined how the terminal-link schedules controlled initial-link response allocation and responding on no-food trials. The top panel of Figure 3.2 shows the logarithm of the ratio of initial-link responses (left/right) for the six blocks of 12 cycles that occurred each session, averaged across subjects. To assess strength of preference, we applied a generalized-matching model (Baum, 1974; Davison and McCarthy, 1988) in which response allocation is a linear function of the log immediacy ratio:

$$\log\left(\frac{B_L}{B_R}\right) = a \log \frac{1/D_L}{1/D_R} + \log b$$

Equation 3.1

In Equation 3.1, B represents response rate and D delay to reinforcement signaled by terminal-link onset. Subscripts L and R refer to left and right alternatives. The slope and intercept of the linear relation, a and log b respectively, represent sensitivity of response allocation to log immediacy ratio and 'bias', a constant preference for one alternative independent of the immediacy ratio.

For each subject, we calculated six log initial-link response ratios per session – one for each block of cycles. We regressed log response ratios on log immediacy ratios to obtain parameter estimates of sensitivity and bias in Equation 3.1 for each block. Bias was not systematically positive or negative across subjects and blocks. Averaged across subjects, sensitivity values (with standard error) were 0.30 (0.05), 0.91 (0.07), 1.11 (0.04), 1.19 (0.01), 1.24 (0.04) and 1.28 (0.06) for the first through sixth blocks, respectively. Response allocation favored the shorter terminal link and stabilized approximately halfway through each session, consistent with previous research. Average sensitivity in blocks 4-6 (1.24) indicates a response ratio of 2.36:1 for the FI 10-s alternative.

To check for effects of immediacy ratios from prior sessions, log response ratios from blocks 4-6 of each session were regressed on log immediacy ratios from that session (Lag 0) and the two previous (Lags 1-2) for individual subjects. For all subjects, estimates of sensitivity coefficients for Lag 0 immediacy ratios were positive, statistically significant, and greater than 1, indicating overmatching, whereas estimates for higher lags were neither systematically positive or negative nor statistically significant. These results suggest there was no effect of immediacy ratios from prior sessions on response allocation.

The bottom panel of Figure 3.2 shows average start and stop times on terminal-link no-food trials for each block and FI schedule. We entered individual average start and stop times into a repeated-measures ANOVA and found significant effects of schedule, $F(1,3) = 32.81$, $p < .05$, and block, $F(5,15) = 18.43$, $p < .001$, as well as a schedule x block interaction, $F(5,15) = 24.69$, $p < .001$. Results for stop times were similar, with significant effects of schedule, $F(1,3) = 100.53$, $p < .01$, block, $F(5,15) = 46.82$, $p < .001$ and their interaction, $F(5,15) = 42.04$, $p < .001$. Post hoc tests (Tukey honestly significant difference) indicated that start times for FI 10- and FI 20-s schedules did not differ in the first block and that start times for FI 10 s did not change over the course of the session, whereas those for FI 20 s became significantly greater across blocks. Stop times for the two schedules were different in the first block and did not change for FI 20 s after the second block, whereas stop times for FI 10 s decreased every subsequent block.

Averaged across subjects, mean start time from the three no-food terminal links in blocks 4-6 was 3.10 ($SE = 0.84$) for FI 10 s and 9.07 ($SE = 1.72$) for FI 20 s. Mean stop times from blocks 4-6 were 13.40 ($SE = 1.64$) and 25.34 ($SE = 1.62$) for FI 10 s and FI 20 s, respectively.

3.2.4.3 Acquisition of control by initial-link duration

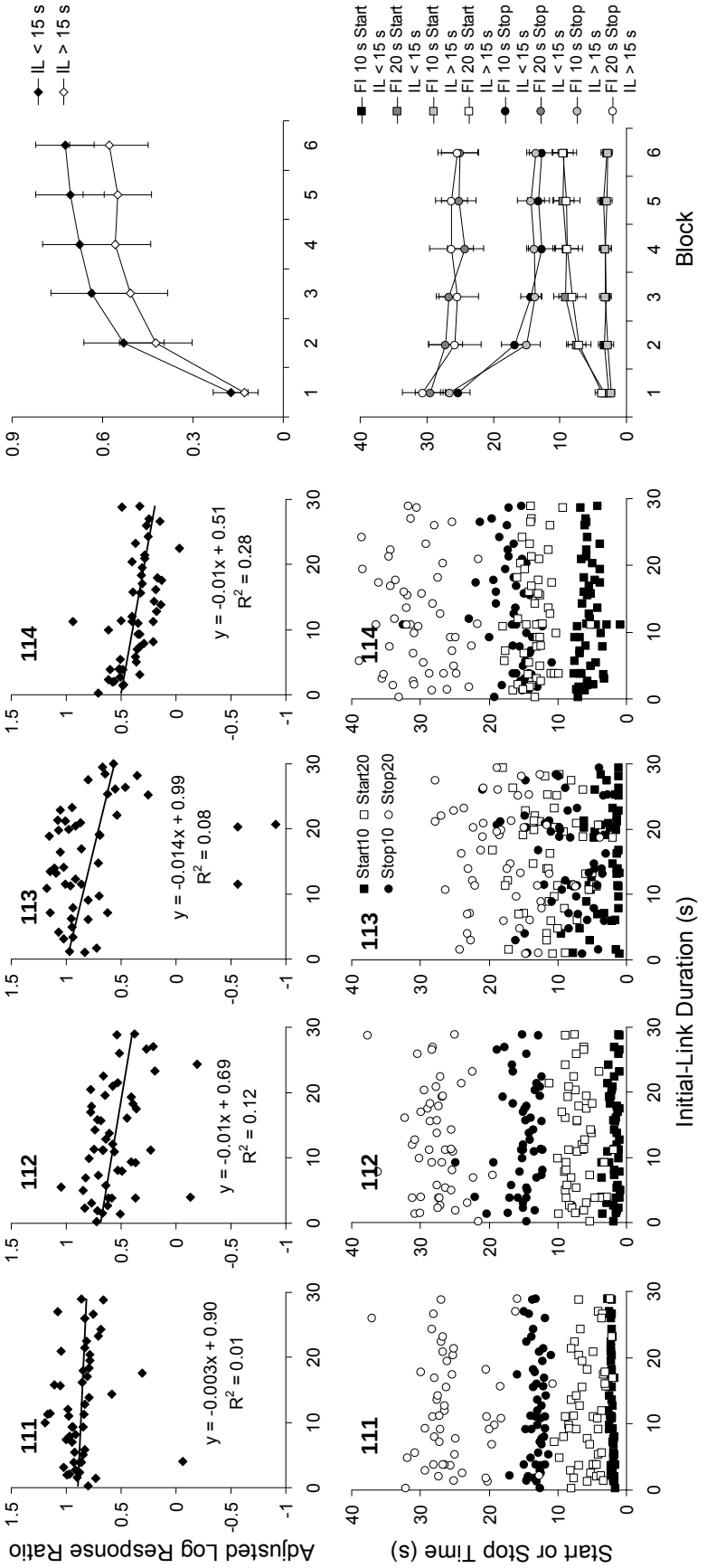

Figure 3.3 (facing page). The four left top panels show adjusted log response ratios as a function of programmed initial-link duration for all subjects. Each data point represents preference for the FI 10 s terminal link relative to an individually calculated indifference point (see text for further explanation) from blocks 4-6 of a single session. Parameters and variance accounted for (R2) by linear regression (solid lines) are also shown. • The rightmost top panel shows group mean adjusted log response ratios as a function of session block. Filled data points represent adjusted log response ratio from blocks for those sessions in which programmed initial-link VI schedule value was < 15 s. Open data points represent equivalent data for sessions in which programmed initial-link VI schedule value was > 15 s. Bars show standard error. Note that the ordinate scale for group mean data is different from the ordinate scale for individual-subject data. • The four left lower panels show start times (squares) and stop times (circles) from no-food trials of FI 10 s (filled data points) and FI 20 s (open) terminal links as a function of programmed initial-link duration for all subjects. Each data point represents mean start or stop time from no-food trials of blocks 4-6 for a single session. • The rightmost lower panel shows group mean start times (squares) and stop times (circles) as a function of session block. Each data point represents the group mean start or stop time for a particular block, terminal-link schedule and short (< 15 s) or long (> 15 s) initial link duration. Fill colors denote different terminal-link schedules and programmed initial-link durations: black for FI 10 s terminal links when programmed initial-link duration was < 15 s; dark gray for FI 20 s terminal links, initial-link duration < 15 s; light gray for FI 10 s terminal links, initial-link duration > 15 s; and white for FI 20 s terminal links, initial-link duration > 15 s. Bars represent standard error.

Next, we examined the relationship between initial-link duration and measures of choice and timing. $\text{Log}(B_L/B_R)$ is a measure of preference for the left terminal link relative to indifference, that is, equal numbers of responses to left and right initial links. We adjusted log response ratios so they reflected subjects' preference for the FI 10-s terminal link relative to an individually-calculated indifference point. This was accomplished by subtracting *log b* (when Equation 3.1 was applied to individual-subject response ratios over blocks 4-6 or single blocks of all 50 sessions) from each log response ratio and then multiplying by -1 those ratios from sessions in which the log immediacy ratio was negative. In this way, we obtained measures of preference corrected for individual bias and for which greater values indicated more extreme response allocation.

The four left top panels of Figure 3.3, labeled 111, 112, 113 and 114, show adjusted log response ratios based on responding in blocks 4-6 plotted as a function of programmed initial-link duration. The equations in each panel describe best-fitting regression lines. Although regression slopes were small, all were negative and were

significant for Pigeons 112 and 114. For data pooled across subjects, there was a significant negative correlation between programmed initial link duration and adjusted log response ratio, $r = -0.20$, $p < 0.01$. Across subjects, there was a small but consistent initial-link effect; shorter initial-links were associated with stronger preference for the FI 10 s terminal link.

To check for hysteresis effects of initial-link duration from prior sessions, adjusted log response ratios from blocks 4-6 of each session were regressed on Lags 0-2 initial link durations for individual subjects. No estimates of sensitivity coefficients for higher lags were statistically significant, which indicates that there was no effect of initial link durations from prior sessions on response allocation in the current session.

Table 3.1. Results of regressing adjusted log response allocation (averaged over the final 3 blocks of each session) on linear and quadratic components of centered Lag 0 programmed initial-link duration. Reported coefficients are unstandardized weights.
*** p < 0.05, ** p < 0.01, *** p < 0.001**

Pigeon	Linear	Quadratic	Intercept	R^2
111	-0.004	0.000	0.901***	0.09
112	-0.009**	0.000	0.613***	0.20
113	-0.012***	-0.001***	0.956***	0.38
114	-0.012***	0.001*	0.327***	0.34
Pooled	-0.008***	0.000	0.690***	0.07

A goal of these analyses was to determine whether the bitonic initial-link effect reported by Christensen and Grace (2008) would be obtained when initial-link duration varied unpredictably across sessions. Table 3.1 shows parameter estimates of a polynomial regression of adjusted log response ratio, blocks 4-6, on linear and quadratic components of centered Lag 0 programmed initial-link duration. If the initial-link effect is bitonic in the direction reported in Christensen & Grace (2008), then the quadratic component should be negative and significant. For data pooled across subjects, there was a significant negative linear coefficient but the quadratic coefficient was not significantly different from zero.

These results indicate that response allocation was weaker for longer absolute initial-link durations, but there was no evidence of a bitonic function.

To characterize the magnitude of the relationship between response allocation and initial-link duration, the rightmost top panel of Figure 3.2 shows the mean adjusted log response ratio for all subjects for each session block, separately for sessions in which programmed initial-link duration was greater than or less than 15 s. We entered adjusted log response ratios from individual subjects into a repeated-measures ANOVA and found a main effect of initial-link duration, $F(1,3) = 14.21$, $p < .05$. This provides additional confirmation of the initial-link effect: Response allocation was more extreme in sessions with programmed initial-link durations less than 15 s. Subjects responded consistently in initial links, emitting a minimum of 33 and a mean of 297 responses per block, thus it is unlikely response ratios in the shortest initial links were affected by a limited opportunity to respond. A main effect of block, $F(5,15) = 34.61$, $p < .001$ indicated preference for the shorter terminal link became more extreme over successive blocks. The interaction was also significant, $F(5,15) = 3.20$, $p < .05$, indicating that the magnitude of the difference in preference for short vs long initial links increased over the course of the session. Because performance was stable in the second half of the session, adjusted log initial-link response ratio from blocks 4-6 were averaged to provide an overall measure of preference. Averaged across subjects, the adjusted log response ratio was 0.70 ($SE = 0.11$) when the initial link was < 15 s and 0.56 ($SE = 0.12$) when the initial link was > 15 s. We calculated the difference between these values, 0.14, as an estimate of the magnitude of the initial-link effect.

The four left lower panels of Figure 3.3, labeled 111, 112, 113 and 114, show mean start and stop times for FI 10 s and FI 20 s alternatives from no-food trials in blocks 4-6, plotted as a function of initial-link duration. Although control by terminal-link delay was greater for some subjects than others, no systematic effects of initial-link duration were apparent. We regressed start and stop times, pooled across subjects, on programmed initial-link duration. No estimated sensitivity coefficients were statistically significant, which suggests that across subjects there was no effect of initial-link duration on start or stop times.

The rightmost lower panel of Figure 3.3 shows group mean start and stop times for both schedules and each session block. Separate values were obtained for sessions in which initial-link duration was shorter or longer than 15 s. We entered start and stop times from individual subjects into a repeated-measures ANOVA with terminal-link schedule, programmed initial-link duration (</> 15 s) and block as factors. For start and stop times, there were significant main effects of terminal-link schedule and block, and a significant terminal-link schedule x block interaction, as reported in section 3.1. No main effects or interactions involving initial-link duration were significant. Unlike initial-link response allocation, start and stop times on no-food trials were determined by the terminal-link schedules exclusively, with no effect of initial-link duration.

3.2.5 Discussion

The goals of this experiment were to characterize how response allocation and temporal control of terminal-link responding in a rapid acquisition concurrent-chains procedure was affected by unpredictable changes in programmed initial-link duration across sessions. We asked whether control by the current session immediacy ratio was similar to

that obtained in previous studies; whether control by initial-link duration evidenced a bitonic function and hysteresis, as reported by Christensen and Grace (2008); and whether there was any effect of initial-link duration on timing.

3.2.5.1 Comparison of timing data with previous research

Start and stop times for FI 10 and FI 20 s terminal links, as well as the degree to which each changed over successive blocks, were comparable to those reported by Kyonka and Grace (2007), who used a similar procedure except that the initial link was always VI 10 s. That temporal control was similar in both experiments in spite of differences in initial-link schedules suggests that start and stop times on no-food trials are not affected by initial-link duration.

3.2.5.2 Comparison of choice data with previous research

In a reanalysis of steady-state experiments that used FI terminal links, Grace (1994) reported an average sensitivity of 2.68 (SE 0.45). Sensitivity values reported in rapid acquisition experiments in which initial-link duration did not change (Kyonka & Grace, 2007) or changed systematically (Christensen & Grace, 2008) were within one standard error of this value. By contrast, the average sensitivity value in our experiment (1.24) was 3.18 standard errors below the value reported by Grace (1994). Sensitivity coefficients stabilized by the second half of sessions and there was no systematic effect of higher-lag immediacy ratios, consistent with previous research.

We found a small but consistent initial-link effect as shown by a negative correlation between adjusted log response allocation, a bias-corrected measure of preference for the FI 10 s terminal-link, and initial-link duration. Adjusted log response ratios were significantly greater when programmed initial-link duration was < 15 s than when it was > 15 s, and the magnitude of that difference increased over the course of the

session. On average, the difference between adjusted log response ratios for < 15 s vs > 15 s initial links was 0.14 for responding in the second half of the session. Note that because initial-link schedule values were sampled from a uniform distribution with a range of 0.01 s - 30 s, the average programmed durations were approximately 7.5 s and 22.5 s, respectively, for sessions in which the initial link was less or greater than 15 s, respectively.

To compare magnitude of the initial-link effect observed in this experiment with that reported Christensen and Grace (2008), we obtained adjusted log response ratios from their study when the programmed initial link schedule was VI 7.5 s: 0.86 (*SE* = 0.07) and VI 22.5 s: 0.64 (*SE* = 0.07). To compare magnitudes of initial-link effects, we entered individual subjects' adjusted log response ratios for short and long initial links from Christensen and Grace's and the current experiment into a repeated-measures ANOVA with initial-link duration and experiment as factors. Although preference was stronger with short initial links than with long, $F(1,9) = 43.84$, $p < .001$, there was no significant effect of experiment or initial-link x experiment interaction, $F(1,9)$'s = 1.12 and 1.81, respectively, both *ns*.

Christensen and Grace reported a hysteresis effect in which initial-link duration from prior sessions influenced response allocation in the current session. In their experiment, hysteresis appeared as a horizontal displacement of the function relating response ratio to initial link duration for ascending vs descending parts of the sequence. Any hysteresis effects in our experiment would appear as a significant sensitivity coefficient for higher-lag initial-link durations. However, none were found. That initial-link duration from previous sessions affected response allocation when changes in initial-

link duration were systematic but not when they were unpredictable could be considered adaptive.

Grace and McLean (2006) described a decision model in which initial-link response strength increases as a function of the probability the preceding terminal-link delay was categorized 'short', and decreases as a function of the probability it was categorized 'long,' relative to a criterion:

$$RS_{n+1} = RS_n + p_{short} * (Max_{RS} - RS_n) * \Delta - (1 - p_{short}) * (RS_n - Min_{RS}) * \Delta$$

Equation 3.2

Expected initial-link response strength for an alternative after the subject has experienced that terminal link *n* times, RS_{n+1}, is determined by RS_n, previous response strength, Max_{RS} and Min_{RS}, maximum and minimum possible response strengths, p_{short}, the probability that the just-experienced terminal-link delay was perceived at short, and Δ, a learning rate parameter. Relative initial-link responding is assumed to equal relative response strength. The value of p_{short} is calculated as the probability that the log terminal link delay is less than a randomly-selected log delay from a normal distribution specified by a mean, referred to as the criterion, and standard deviation, σ.

In Christensen and Grace's extended version of the decision model, the comparison distribution represented the history of delays between stimuli experienced by the subject. They showed that if the criterion was determined by the intervals between the onset of the initial links and terminal links, as well as those between the terminal links and reinforcement, the model predicts a bitonic relationship between response allocation and initial-link duration.

One possible way to reconcile the absence of evidence of a bitonic initial-link function in this experiment with results of Christensen and Grace is to note that there was greater variability in initial-link duration across sessions. To the extent that initial-link intervals contribute to the criterion, this increased variability might have reduced the accuracy with which delays were judged short or long. The primary mechanism in the model for reducing accuracy is to increase the standard deviation parameter.

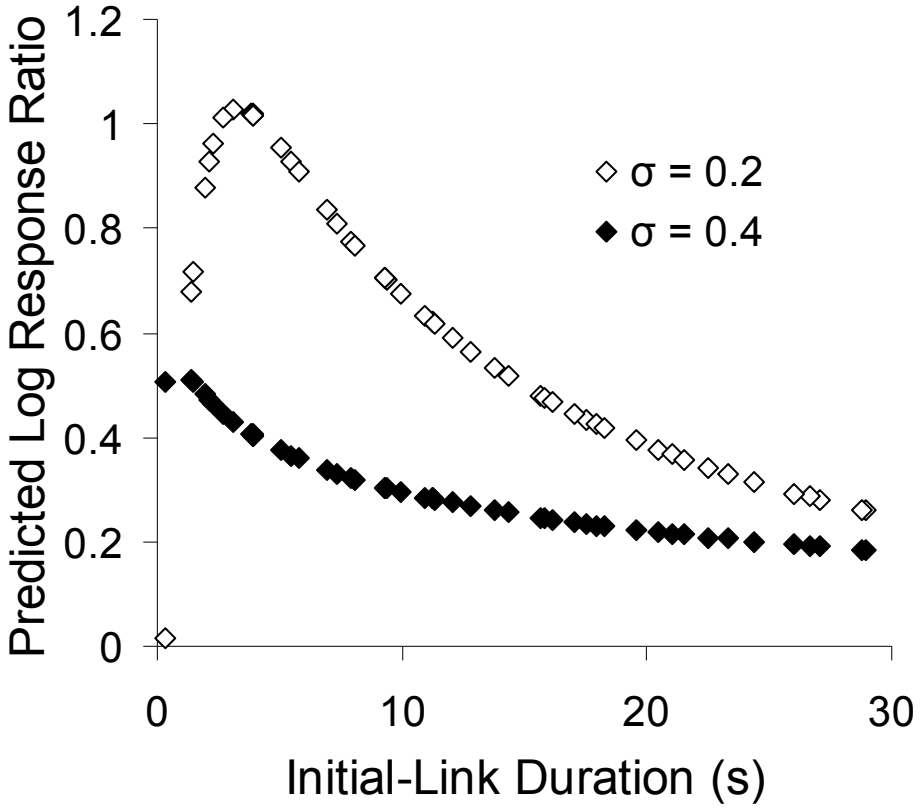

Figure 3.4. Log initial-link response allocation predicted by the extended decision model using a relatively small (open data points, σ = 0.2) and large (filled data points, σ = 0.4) standard deviation for the criterion. For both series, Max_{RS} = 1, Min_{RS} = 0.01, Δ = 1 and p_{short} = 1 – Φ(log *D*, log *C*, σ), where Φ is the cumulative normal distribution with a mean of log *C* and standard deviation σ, evaluated for the preceding terminal link delay (log *D*). The criterion log *C* is determined each session as (log D_L + log D_R + 2*log *IL*)/4, where D_L and D_R are left and right terminal-link delays and *IL* is the initial-link schedule value.

Figure 3.4 shows predicted adjusted log response ratios derived from the extended decision model. Open data points show predictions based on parameters identical to those

used in Christensen and Grace's Figure A1. Filled data points show predictions when all parameters are identical, with the exception of a greater standard deviation. All else being equal, a greater standard deviation attenuates overall strength of preference and mitigates the bitonic element of the initial-link effect. Thus, it is possible that a reduction in discrimination accuracy might account for the observed differences between our experiment and Christensen and Grace (2008). Nevertheless, the present results show that when initial-link duration is changed unpredictably across sessions, the effect on preference is similar to that observed in steady-state studies. Although the decision model requires clarification and development in terms of how the criterion is updated, and what variables might affect the standard deviation and hence decision accuracy, it has potential to provide an integrated account of concurrent-chains choice under both dynamic and steady-state conditions.

4 Chapter 4 Rapid acquisition of choice and timing: Effects of terminal-link duration

4.1 Notes on Experiment 3

Chapter 4 presents Experiment 3, which is in preparation as a manuscript for future publication. Four pigeons responded in a maximal-variation concurrent chains plus peak procedure in which pairs of terminal-link delays were either short or long. The reasons for manipulating terminal-link duration were to replicate results from the minimal-variation condition of Experiment 1 using the same range of immediacy ratios but different delays, and to determine how absolute terminal-link duration affected initial- and terminal-link performance. Christensen and Grace (2009a) presented results of a minimal-variation rapid acquisition concurrent chains experiment in which terminal-link delays were short or long in different conditions. They reported that pigeons' sensitivity to log immediacy ratio was greater when delays were long than when they were short, consistent with the terminal-link effect (MacEwen, 1972). They applied a version of Grace and McLean's (2006) decision model that predicted effects of absolute terminal-link duration. Application of a decision model to this data is presented in Chapter 6. Experiment 3 differs from Christensen and Grace's (2009a) experiment in several ways, notably the inclusion of no-food trials and the distribution of programmed delays. Whereas comparing short and long pairs of delays in minimal-variation concurrent chains can determine whether absolute terminal-link duration affects performance, comparing short and long maximally varying pairs of delays could shed light on how absolute terminal-link duration affects performance.

4.2 Experiment 3: Effects of absolute terminal-link duration on choice and timing performance when immediacy ratios change unpredictably across sessions

4.2.1 Abstract

Four pigeons responded in a concurrent-chains procedure in which terminal-link schedules changed pseudorandomly across sessions. Schedules were either overall long, delays summed to 45 s, or overall short, delays summed to 15 s. Across sessions, the location of the shorter terminal-link changed according to a pseudorandom binary sequence. On some terminal links, food was withheld to obtain start and stop times, measures of temporal control. Log terminal-link immediacy ratio controlled log initial-link response ratios, which stabilized within the first half of each session. They fell into two clusters determined by whether the subject preferred the left or right alternative. Within-cluster sensitivity, assessed by a piecewise linear model, was similar across clusters and terminal-link durations. Sensitivity to relative immediacy was greater when delays were overall long than when they were overall short, replicating the terminal-link effect. Temporal control of stop times was similar in short and long sessions, but temporal control of start times was relatively more precise in long than sessions. Results extend our understanding of the terminal-link effect that response allocation is more extreme with longer delays to choice in transition with a different pair of terminal-link delays in effect each session.

148. TERMINAL-LINK DURATION, CHOICE AND TIMING

Keywords: concurrent chains, terminal link effect, rapid acquisition procedure, temporal control, pigeons

4.2.2 Introduction

In typical concurrent-chains procedures (Herrnstein, 1964), subjects respond to two concurrently-presented signaled options or 'initial links' that operate on a variable interval (VI) schedule of reinforcement. Responding in initial links produces either of two mutually-exclusive outcomes or 'terminal links' that each end with food reinforcement after another reinforcement schedule has been satisfied. Traditional concurrent chains studies have used designs in which the same terminal-link schedules operate for many sessions. The usual result is that subjects respond more to the initial link preceding the terminal link associated with the relatively shorter delay to reinforcement (see Mazur, 2001, for review). McCarthy and Davison (1988) quantified the relationship between preference, measured as initial-link response allocation, and relative terminal-link immediacy (i.e., the reciprocal of delay) as linear:

$$\log\left(\frac{B_L}{B_R}\right) = \log b + a \ \log\left(\frac{1/D_L}{1/D_R}\right).$$

Equation 4.1

In Equation 4.1, B is response rate, D is delay to reinforcement from terminal-link onset, and subscripts L and R denote left and right alternatives, respectively. Equation 4.1 states that log initial-link response ratio is a linear function of log immediacy ratio with slope a and intercept log$b$. The slope and intercept parameters are often referred to as sensitivity and bias, respectively, because the slope indicates how sensitive response ratio is to differences in immediacy ratio, and the intercept represents a constant preference for a

particular alternative that is independent of immediacy ratio. Applied to individual experiments, Equation 4.1 generally provides an accurate account of initial-link behavior, accounting for over 90% of the variance in response ratios from concurrent-chains experiments (Grace, 1994).

Preference in steady state concurrent chains can be affected by other temporal factors, including absolute duration of terminal links. MacEwen (1972) presented pigeons with concurrent-chains schedules with fixed-interval (FI) or variable-interval (VI) terminal links in which the longer delay was always double the shorter scheduled delay, which ranged from 5-40 s across experimental conditions. MacEwen reported the 'terminal-link effect' that longer absolute terminal-link schedule values intensified preference. Williams and Fantino (1978) replicated the terminal-link effect and showed that it could be predicted by Delay-Reduction Theory (Squires & Fantino, 1971), which posits that the value of terminal links is determined by the reduction in delay to primary reinforcement, relative to the overall average time between initial-link onset and reinforcer delivery, signaled by onset of a terminal link. Other steady-state models of choice (Grace, 1994; Mazur, 2001) differ in their quantitative details but similarly predict greater sensitivity to immediacy ratio with longer terminal-link delays.

Theories of concurrent-chains choice assume that sensitivity to relative immediacy is greater with longer absolute terminal-link delays because absolute delays (as well as relative immediacy) determine relative value. An alternative possibility is that greater sensitivity with longer delays reflects greater discriminability rather than a difference in the subject's valuation of each terminal link. Investigating concurrent-chain performance in transition could distinguish between these possibilities. One approach to investigating

choice in transition is to present subjects with pairs of terminal-link delays in which the left:right location of the initial-link alternative associated with the shorter terminal-link delay changes unpredictably across sessions. In concurrent-chains experiments that employ this "rapid acquisition" approach (Christensen & Grace, 2008, 2009a; Grace et al., 2003, Grace & McLean, 2006, Kyonka and Grace, 2007, 2008, 2009), pigeons' initial-link response allocation adapts rapidly: responding typically stabilizes at or before half of 72 initial- and terminal-link cycles have elapsed and is determined by relative immediacy in the current but not previous sessions.

In one condition of Kyonka and Grace's (2007) experiment, terminal-links were reinforced according to FI schedules. Immediacy ratios were drawn from a uniform distribution, the location of the shorter terminal link varied across sessions according to a pseudorandom binary sequence and the sum of left and right schedule values from the same session was always 30 s. One sixth of terminal links were "no food" trials that ended after 60 s without reinforcement. Responding in no food trials provided measures of temporal control (cf. Cheng and Westwood, 1993). Kyonka and Grace found that, once pigeons had experience with the procedure, all measures of preference and temporal control stabilized approximately halfway through sessions with no evidence of influence of prior sessions. Whereas initial-link response allocation in steady-state procedures is widely regarded as a linear function of relative immediacy, Kyonka and Grace reported that response ratios tended to fall in two clusters, consistent with a partially categorical or winner-take-all mechanism. By contrast, measures of temporal control were linear functions of terminal-link delays with no evidence of nonlinearity or influence of previous sessions. In spite of the difference in functional relationships between dependent and independent variables,

measures of choice and temporal control covaried beyond variance attributable to terminal-link immediacy ratio.

Grace and McLean (2006) proposed a quasidynamic decision model to describe concurrent-chains choice in transition and steady-state. Their decision model proposed a linear-operator (Bush & Mosteller, 1956) process by which response strength increases if the just-experienced terminal-link delay is categorized as short relative to a comparison distribution, and decreases if it is categorized as long. Grace and McLean assumed a single comparison distribution applied to both terminal-link alternatives and that log initial-link response ratios reflect relative expected response strength:

$$\log\left(\frac{B_L}{B_R}\right) = \log\left(\frac{RS_L}{RS_R}\right),$$

Equation 4.2

where

$$RS_{n+1} = RS_n + p_{short} * \left(Max_{RS} - RS_n\right) * \Delta - \left(1 - p_{short}\right) * \left(RS_n - Min_{RS}\right) * \Delta .$$

Equation 4.3

RS_{n+1} is response strength after the subject has experienced the terminal link n times. Min_{RS} and Max_{RS} denote minimum and maximum possible response strengths, respectively. The proportion of the comparison distribution that is longer than the terminal-link delay determines p_{short}, the probability that delay is categorized as short. The probability a delay is categorized as long is 1 - p_{short}. The learning rate parameter Δ determines how quickly responding stabilizes with experience. Grace and McLean showed that the model could describe the linear relationship between log response and immediacy ratios observed in steady state experiments as well as the relatively categorical relationship observed when delays change unpredictably each session.

Christensen and Grace (2009a) used a rapid acquisition concurrent-chains procedure to determine whether an extension to the decision model could predict effects of absolute terminal-link duration when immediacy ratio changed unpredictably across sessions. There were two conditions: in the short condition, the left terminal link was always reinforced according to an FI 8-s schedule. The right terminal-link FI schedule value was either 4 or 16 s. In the long condition, left and right FI schedule values were 16 and either 8 or 32 s. Sensitivity to relative immediacy was greater in the long than the short condition. Christensen and Grace applied the decision model to within-session log initial-link response ratios from both conditions. They predicted that the mean of the criterion distribution that determines p_{short} was a function of the average initial- and terminal-link delays experienced in the condition, so they applied a single set of decision model parameters to responding from short and long conditions but estimated separate criterion means for each condition. As predicted, estimated criterion means were larger in the long condition than the short for three out of four subjects. Christensen and Grace concluded that the decision model "provided a good description of the data and predicted the terminal-link effect."

In the present experiment, terminal-link schedule values changed pseudorandomly each session. The delays associated with left and right terminal links either summed to 15 s (i.e., they were Short) or to 45 s (Long). Our primary goal was to characterize effects, if any, of absolute terminal-link duration on preference for the shorter delay to reinforcement and on measures of temporal control. In particular, we wanted to determine: 1) whether there would be a categorical-type relationship between log response and immediacy ratios, as observed in similar experiments (Grace & McLean, 2006; Kyonka & Grace, 2007). 2) What is the effect (if any) of terminal-link duration on choice in transition when schedule

values changed unpredictably as well as immediacy ratios: is sensitivity to immediacy ratio greater when delays are Long than when they are Short? 3) If the categorical-type relationship and the terminal-link effect occurs, does the difference in sensitivity affect responding within or between categories? 4) What are the effects (if any) of terminal-link duration on temporal control?

Finally, an overarching goal of this experiment is to compare within-session acquisition of preference in sessions with short and long delays. Pigeons' adaptation to unpredictably-changing delays may make it possible to distinguish or disconfirm whether discriminability or valuation produces the terminal-link effect. If the difference in sensitivity is attributable to greater discriminability of longer delays, within-session acquisition of preference should be faster for long delays than for short. However, if the difference in sensitivity is attributable to a difference in relative value, there may be no difference in rate of acquisition. Assuming response allocation data falls into clusters consistent with categorical discrimination, if differences in relative value produce the terminal-link effect, differences in sensitivity should occur within categories rather than between categories.

4.2.3 Method

4.2.3.1 Subjects

Four pigeons of mixed breed and sex, numbered 221-224, were maintained at 85% *ad libitum* weight plus or minus 15g through appropriate post-session feedings. Pigeons were housed individually in a vivarium with a 12-hr:12-hr light:dark cycle plus windows providing natural light and with free access to water and grit. All had experience with

concurrent-chains procedures in which terminal-link delays changed unpredictably across sessions.

4.2.3.2 Apparatus

Four operant chambers (32 cm deep x 34 cm wide x 34 cm high) were enclosed in sound-attenuating boxes containing ventilation fans. Each chamber contained three keys 21 cm above the floor arranged in a row 10 cm apart, a houselight located above the center key, and a grain magazine with a 5 x 5.5 cm aperture that was centered 6 cm above the floor. The houselight provided general illumination at all times except during reinforcer delivery. The magazine, which was illuminated during reinforcement, contained wheat. A force of approximately 0.15 N was necessary to operate each key. Experimental events were controlled through a microcomputer and MED-PC® interface located in an adjacent room.

4.2.3.3 Procedure

Because all pigeons had previous experience, training began immediately in the concurrent-chains procedure. Sessions ended after 72 initial- and terminal-link cycles or 90 minutes, whichever came first.

At the start of a cycle, side keys were lighted white to signal initial links. A terminal-link entry was assigned pseudorandomly to the left or right key with the constraint that in every block of 12 cycles, 6 were assigned to each key. If an interval selected from the initial link schedule had timed out and it satisfied a 1-s changeover delay, a response to the preselected key produced a terminal link entry.

The initial-link schedule did not begin timing until the pigeon first pecked either key. In this way, pausing after the completion of terminal links was excluded from initial-

link time. The initial-link schedule was VI 10 s and contained 12 intervals sampled without replacement and constructed from geometric progression (Fleshler & Hoffman, 1962).

Terminal-link entry was signaled by extinguishing the side keys and lighting the center key. The color of the center key depended on whether a left or right initial-link response had produced the terminal link (red – left, green – right). Of the six terminal-link entries that were scheduled in a block for each alternative, the computer pseudorandomly assigned five to be food trials and one to be a no-food trial. On food trials, pecks to the center-key were reinforced with 3 s access to grain according to an FI schedule. A 5-s limited hold was in effect, such that if a response was not made within 5 s after the FI schedule had elapsed, the terminal link ended and no reinforcement was delivered. On no-food trials, the center key was lighted red or green for 90 s and there was no consequence of a center key peck at any point in the interval. For both types of trials, after a terminal link ended the side keys were lighted white signaling the initial links and the beginning of the next cycle.

The pair of FI schedule values in effect for each session was sampled from a potentially infinite population of values. The location of the shorter terminal-link delay was changed across sessions according to a 31-step pseudorandom binary sequence similar to the one used by Hunter and Davison (1985). The expected log immediacy ratios (i.e., log[1/DelayL / 1/DelayR]) were log(2) and log(1/2), respectively, for sessions in which the terminal-link delay associated with the left and right alternatives were shorter. To accomplish this, we determined the terminal-link schedule values for each session using a pseudorandom number generator subject to the constraints that immediacy ratios be

uniformly distributed between log(1/4) and log(4). Left and right delays summed to 15 s in every 'short' session and to 45 s in every 'long' session.

Measures of temporal control on individual no-food trials were obtained using the method of Cheng and Westwood (1993). Responses from individual no-food trials were sorted into 1-s bins. The time of occurrence of the first response from the first instance of three consecutive filled bins was designated the start time. The time of occurrence of the last response before three consecutive empty bins was designated the stop time.

There were three conditions. In the Short condition, all sessions were short, pairs of left and right terminal-link delays always summed to 15 s. In the Long condition, all sessions were Long sessions and pairs of delays summed to 45 s. In the Mixed condition, a separate 31-step pseudorandom binary sequence determined whether sessions were short or long. Each condition was scheduled to last 50 sessions, but because a new session was only considered to have begun when delays switched from Short to Long (or vice versa), conditions were in effect for 43-54 sessions. Four conditions occurred in the following order: Short-Long-Mixed-Short for Pigeons 221 and 222, and Long-Short-Mixed-Short for Pigeons 223 and 224. Due to computer error, timing data were only recorded for the first two conditions.

4.2.4 Results

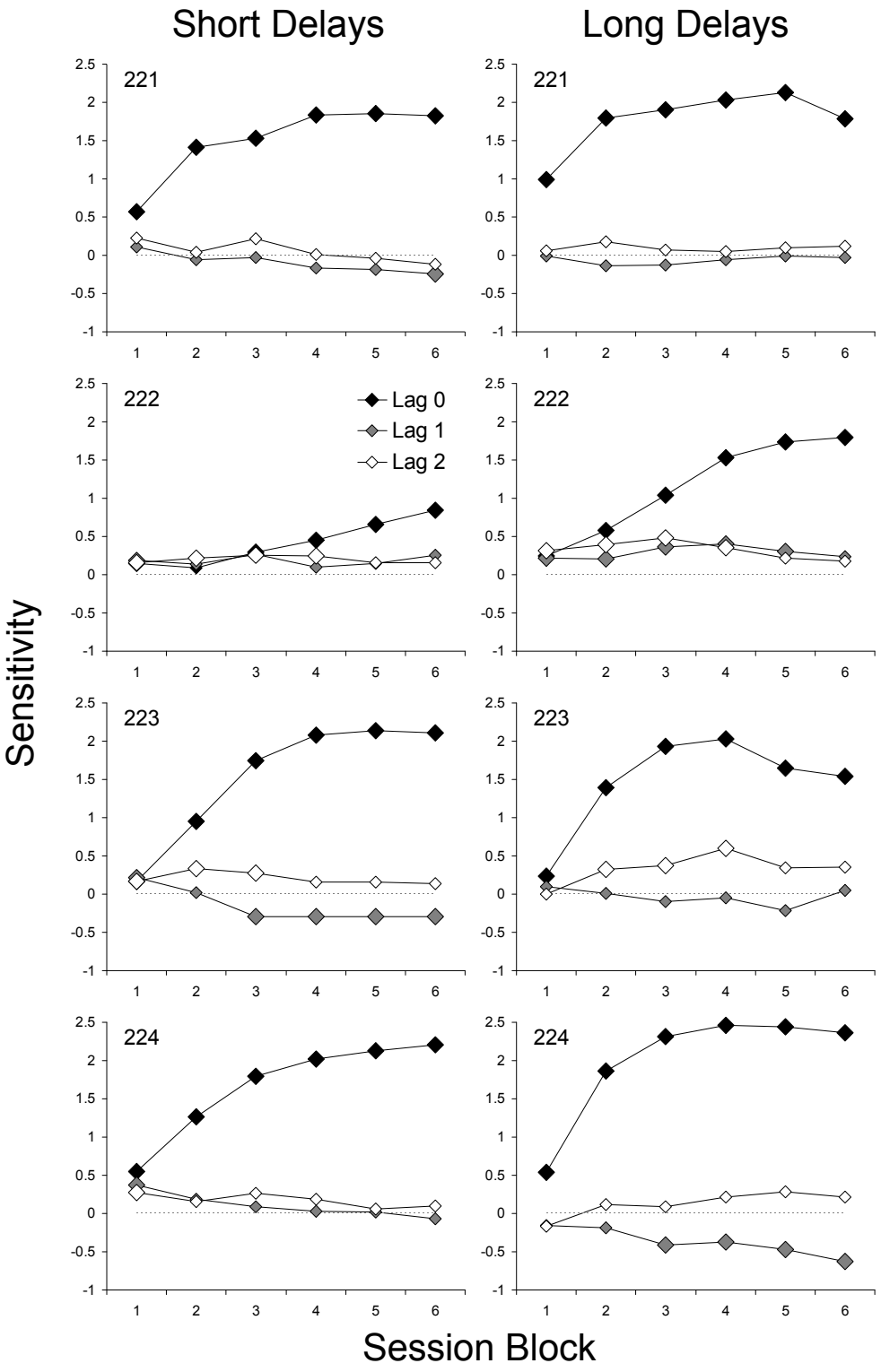

Figure 4.1. Sensitivity of log initial-link response ratios to log immediacy ratios for Lag 0 through Lag 2 for each block of 12 initial- and terminal-link cycles. Larger data points represent parameter estimates that are significantly greater or less than zero.

To determine whether there were effects of log immediacy ratios from previous sessions, we applied a generalized matching model (Baum, 1974; McCarthy & Davison, 1988) that included programmed log immediacy ratios from the current (Lag 0) and three previous (Lags 1-2) sessions as parameters:

$$\log\left(\frac{B_L}{B_R}\right) = \log b + a_0 \log\left(\frac{1/D_{L0}}{1/D_{R0}}\right) + a_1 \log\left(\frac{1/D_{L1}}{1/D_{R1}}\right) + a_2 \log\left(\frac{1/D_{L2}}{1/D_{R2}}\right)$$

Equation 4.4

In Equation 4.4, *B* is initial-link responding, *D* is terminal-link delay to reinforcement, each *a* is a sensitivity coefficient, and log *b* is response bias. The subscripts *L* and *R* refer to the left and right alternatives, respectively, and numerical subscripts refer to session lag. To limit the complexity of the model, only terms up to Lag 2 were included because previous research has generally found no evidence of significant control by higher lags (Grace, Bragason, & McLean, 2003; Grace & McLean, 2006; Schofield & Davison, 1997).

Access to left and right terminal-link reinforcement was equated in blocks of 12 cycles: exactly five food and one no-food left and right cycles are programmed to occur in pseudorandom order in cycles 1-12, 13-24, 25-36, 37-48, 49-60 and 61-72. We calculated log initial-link response ratios using the number of left and right responses made in each block of 12 cycles. Ratios were obtained from all blocks of all sessions of all four conditions and sorted into short and long sessions (that is, short sessions from Short and Mixed conditions were grouped, as were long sessions from Long and Mixed conditions). Log response ratios were regressed on Lags 0-2 immediacy ratios to obtain parameter estimates for Equation 4 for individual subjects. Figure 4.1 shows the resulting sensitivity coefficients for immediacy ratios from the current session (Lag 0) and two previous sessions (Lags 1 and 2). For all subjects, Lag 0 sensitivity coefficients increased over the

first three blocks, whereas Lag 1 and Lag 2 coefficients either did not change systematically or decreased over the course of a session.

For further quantitative characterization of effects of relative immediacy on response allocation, we entered parameter estimates for Lags 0-2 immediacy ratios into a repeated-measures analysis of variance (ANOVA) with terminal-link duration (short or long), session Lag and block as factors. There were main effects of Lag and of block, $F(2,6) = 21.90$, $p<0.01$ and $F(5,15) = 23.60$, $p<0.001$, respectively, but not of terminal-link duration, $F(1,3) = 1.85$, *ns*. There were interactions between terminal-link duration and block, $F(5,15) = 4.29$, $p<0.05$, and Lag and block, $F(10,30) = 19.52$, $p<0.001$, but there was no interaction between terminal-link duration and Lag, $F(2,6) = 2.40$ and no three-way interaction, $F(10,30) = 0.75$, both *ns*. Post-hoc analyses (Tukey Honestly-Significant Difference [HSD]) showed that responding in short and long sessions stabilized after the third block of cycles, consistent with previous research (Grace, Bragason & McLean, 2003; Grace & McLean, 2006; Kyonka & Grace, 2007, 2008, 2009). Although there was no detectable difference in rate of acquisition for short versus long sessions, across subjects response allocation in this experiment replicated the terminal-link effect: Lag 0 sensitivity in the second half of the session was 1.67 [SD = 0.69] when terminal-link delays were short and 1.94 [SD = 0.34] when terminal-link delays were long. Sensitivity to Lag 0 immediacy ratios was greater when terminal-link delays were long than when they were short.

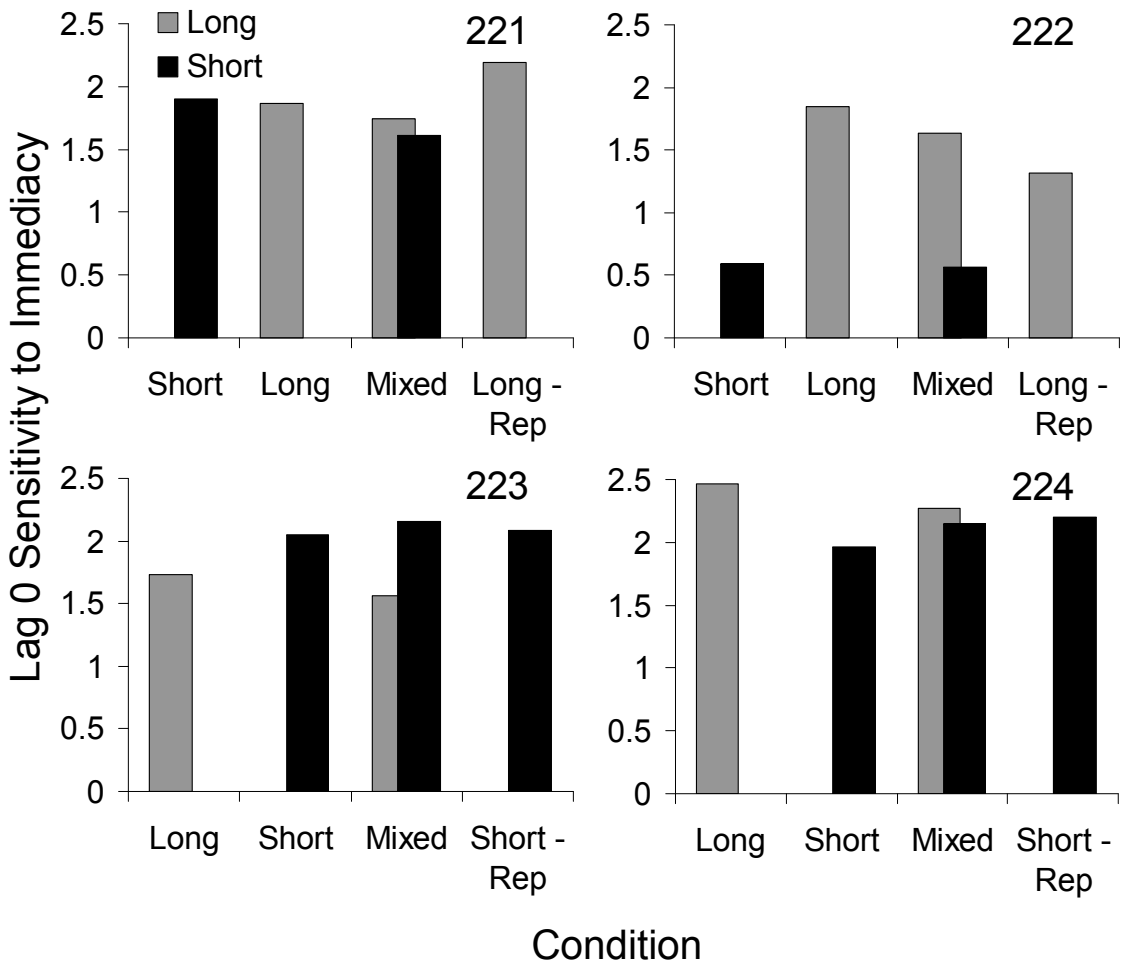

Figure 4.2. Sensitivity coefficients successive conditions. Coefficients were estimated by regressing log initial-link response ratios from blocks 4-6 on Lag 0 log immediacy ratios.

To confirm the presence of the terminal-link effect in this data, we regressed log response ratios, calculated based on responding in blocks 4-6, on Lag 0 immediacy ratios alone and obtained separate estimates of sensitivity to immediacy ratios for sessions in which delays were short and long. Figure 4.2 shows sensitivity coefficients for individual subjects in each condition. The terminal-link effect is present if the grey bars, representing sensitivity from sessions with long terminal-link delays, are higher than the black bars, which represent sensitivity from sessions with short terminal-link delays. This was the case across Short and Long conditions, and for short versus long sessions from the Mixed condition, for all subjects except Pigeon 223. Although the differences in sensitivity

coefficient values across subjects were not significant ($t(3) = 1.05$ and $t(3) = 0.54$, both *ns*, for Short versus Long conditions and short versus long sessions in the Mixed condition, respectively), the terminal-link effect was present for three out of four subjects and quite dramatic for one (Pigeon 222).

Figure 4.2 also shows, at a gross level, no systematic effect of exposure to the Mixed condition on sensitivity when Short or Long delays are presented alone. Overall, sensitivity to short and long delays in the Mixed condition tended to be lower than sensitivity in Short and Long conditions, respectively. Averaged across subjects, sensitivity coefficients were 1.62 [SD = 0.75] and 1.80 [SD = 0.32] for short and long sessions of the Mixed condition, respectively. Equivalent averaged[1] coefficients for Short and Long conditions were 1.66 [SD = 0.72] and 1.95 [SD = 0.39]. However, relative to the initial presentation, sensitivity to Lag 0 immediacy in the replication was greater for one (Pigeon 221) of the two subjects that experienced a replication of the Long condition and lower for the other (Pigeon 222). The same was true for subjects that experienced a replication of the Short condition: sensitivity increased for Pigeon 224 and decreased for Pigeon 223 relative to initial presentation of the Short condition. There was a systematic decrease in sensitivity in the Mixed condition relative to Short and Long conditions, which suggests that, unlike effects of immediacy ratios from previous sessions, effects of absolute terminal-link duration may carry over across sessions. However, the lack of a systematic effect of the Mixed condition on sensitivity in subsequent conditions indicates that any carryover effect did not persist long-term.

[1] To obtain average sensitivity coefficients across subjects, we calculated a single sensitivity coefficient for each subject based on the mean of coefficients estimated for initial presentation and replication of the Long condition for Pigeons 221 and 222, and the Short condition for Pigeons 223 and 224.

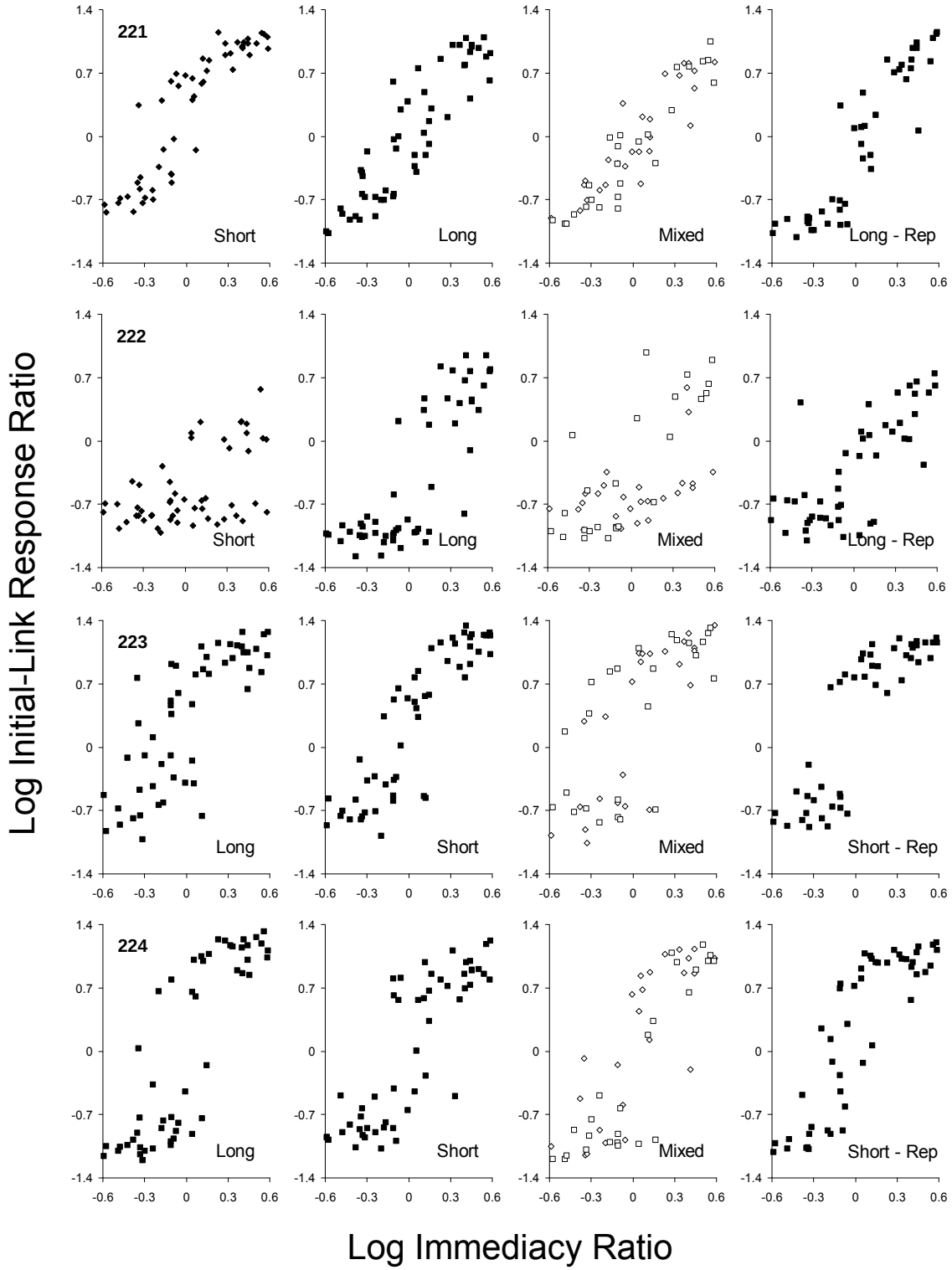

Figure 4.3. Log initial-link response ratios plotted as a function of log immediacy ratios for individual subjects from each condition. Each data point represents performance from blocks 4-6 of a single session. Solid diamonds and squares represent data from Short and Long conditions, respectively. Open diamonds and squares represent data from short and long sessions of the Mixed condition, respectively.

In previous experiments in which terminal-link delays changed unpredictably across sessions (Grace & McLean, 2006; Kyonka & Grace, 2007), initial-link response allocation was a nonlinear function of relative terminal-link immediacy. In particular, Kyonka and Grace (2007) found that log response ratios fell into two clusters depending on whether the left or right alternative was favored. For each condition and individual subjects, Figure 4.3 shows log response ratios (left/right) calculated over the 37th-72nd initial links of individual sessions, plotted as a function of the log ratio of the left vs right terminal-link immediacies in effect for that session. Varying degrees of the clustering reported by Kyonka and Grace are evident in most panels of Figure 4.3.

To confirm deviations from linearity, we compared fits of a linear model with fits of piecewise linear models to log initial-link response ratios from the second half of sessions. For the linear model, we fit separate straight lines to data from sessions with short terminal-link delays and those with long delays. Thus, the linear model had four parameters: a slope and intercept for short sessions and another slope and intercept for long sessions. The piecewise models were adaptations of Kyonka and Grace's (2007) piecewise linear model, in which data were sorted into two groups based on whether the subject had made more responses to the left or right alternative, and the following model with separate linear functions that had the same slope but different intercepts was fitted:

$$if\ \log\left(\frac{B_L}{B_R}\right) < 0 : \log\left(\frac{B_L}{B_R}\right) = \log b_{Right} + a\ \log\left(\frac{1/D_L}{1/D_R}\right)$$
$$if\ \log\left(\frac{B_L}{B_R}\right) \geq 0 : \log\left(\frac{B_L}{B_R}\right) = \log b_{Left} + a\ \log\left(\frac{1/D_L}{1/D_R}\right)$$

Equation 4.5

In addition to the three-parameter piecewise linear model (3-piece) described by Equation 4.5, we fitted a 5-parameter (5-piece) model with four separate lines for data from short and

long sessions in which responding favored the left and right alternatives. All four lines had the same slope. Finally, we fitted a piecewise linear model with 6 parameters (6-piece). The 6-piece model was identical to the 5-piece model except that, like the linear model, there were separate slopes for data from sessions with short and long terminal-link delays.

Table 4.1. Parameter estimates and variance accounted for (VAC) by linear and piecewise linear models.
*** p < 0.05, ** p < 0.01, *** p < 0.001**

	Slope		Intercept		VAC
Linear	Short	Long	Short	Long	
221	0.97	1.04	0.44	-0.06	0.80
222	0.56	1.44	-0.09	0.09	0.56
223	1.11	0.89	-0.02	0.02	0.67
224	0.93	1.07	0.04	-0.04	0.71

	Slope	Intercept		VAC
3-piece		Left	Right	
221	1.14	1.54	-0.44	0.89
222	1.10	0.81	-1.54	0.84
223	0.98	0.75	-0.81	0.93
224	0.84	0.75	-0.75	0.93

		Intercept					
	Slope	Short		Long		VAC	VAC_{inc} relative to
5-piece		Left	Right	Left	Right		3-piece[1]
221	0.95	0.41	-0.27	0.26	-0.41	0.91	0.01***
222	0.81	0.83	-1.03	1.50	-1.31	0.86	0.02***
223	0.88	0.73	-0.76	0.73	-0.69	0.93	0.00
224	0.85	0.74	-0.68	0.83	-0.88	0.93	0.01***

			Intercept							
	Slope		Short		Long		VAC	VAC_{inc} relative to		
6-piece	Short	Long	Left	Right	Left	Right		Linear[2]	3-piece[3]	5-piece[4]
221	0.83	1.10	0.47	-0.31	0.24	-0.40	0.91	0.10***	0.01***	0.002
222	0.48	1.24	1.02	-1.14	1.48	-1.37	0.86	0.30***	0.02***	0.004*
223	0.99	0.85	0.75	-0.78	0.78	-0.74	0.93	0.26***	0.00	0.000
224	0.91	0.88	0.77	-0.72	0.88	-0.94	0.93	0.22***	0.01**	0.000

Significance of increase in VAC determined based on F ratio with (numerator, denominator) degrees of freedom of: 1. (2,191), 2. (2,190), 3. (3,190), 4. (1,190).

Table 3.1 shows parameter estimates (slopes and intercepts) and variance accounted for (VAC) for the linear, 3, 5 and 6 parameter piecewise linear models, as well as

results (*F* ratios) of selected/relevant model comparisons. If the data in Figure 4.3 were linear, there should be no systematic difference between the fitted values for the intercepts (log b_{Right} and log b_{Left}) in Equation 4.5 or any other piecewise model. However, for all subjects and all three piecewise linear models, intercepts for right-preferred clusters were always negative and intercepts for left-preferred clusters were positive, confirming the visual impression that data in Figure 4.3 tended to fall into two separate clusters with much higher sensitivity to relative differences in immediacy between clusters than within, for all subjects.

We compared goodness-of-fit estimates of the 5- and 6-piece models obtained for data from individual subjects with each other and with VAC estimates for the 3-piece model. To two decimal places, VAC of the 6-piece model was equal to VAC of the 5-piece model ($M = 0.91$, $SE = 0.02$); the extra slope parameter in the 6-piece model does not improve the overall goodness of fit. Both 5- and 6-piece models represented significant improvements in VAC over the 3-piece model ($M = 0.90$, $SE = 0.02$) for all subjects except Pigeon 223. We also compared VAC in the 6-piece model with VAC in the 4-parameter Linear model ($M = 0.69$, $SE = 0.05$). The 6-piece and Linear models both feature separate slopes and intercepts for short and long pairs of terminal-link delays, but the 6-piece model also includes separate intercepts for left-preferred and right-preferred clusters. Across subjects, the 6-piece model accounted for an average of 22% more variance than the linear model, and the improvement was significant for all subjects. Taken together, model comparison *F*-ratios indicate that piecewise linear models provide a much better description of initial-link response allocation than linear models in this experiment. Also, of the piecewise linear models examined, the 5-piece model, with a single slope describing within-cluster sensitivity to immediacy and four separate intercepts (for left-preferred and

right-preferred clusters with short and long delays) provides the best account of the piecewise models. Between-cluster sensitivity was greater in long than in short sessions: in other words, the difference between left-preferred and right-preferred intercepts was always greater in long (M = 1.65, SE = 0.44) than short (M = 1.36, SE = 0.25) sessions, although the difference was not significant across subjects, $t(3) = 0.30$, *ns*. However, as the 6-piece model did not improve VAC relative to the 5-piece model, within-cluster sensitivity did not differ across left versus right clusters or as a function of terminal-link duration.

One of the aims of this experiment was to determine whether terminal-link duration affected within-session acquisition of temporal control. The unpredictably-changing procedure used in this experiment is particularly suited to addressing this issue because subjects experience a different pair of delays each session and responding tends to stabilize within a session. To quantify the relationship between scheduled terminal-link delay and temporal control, we regressed start and stop times from each block on the terminal-link delay in effect for that alternative in that session. Exactly one left and one right no-food terminal link occurred in each block. We calculated separate regression lines for start and stop times from Short and Long conditions.

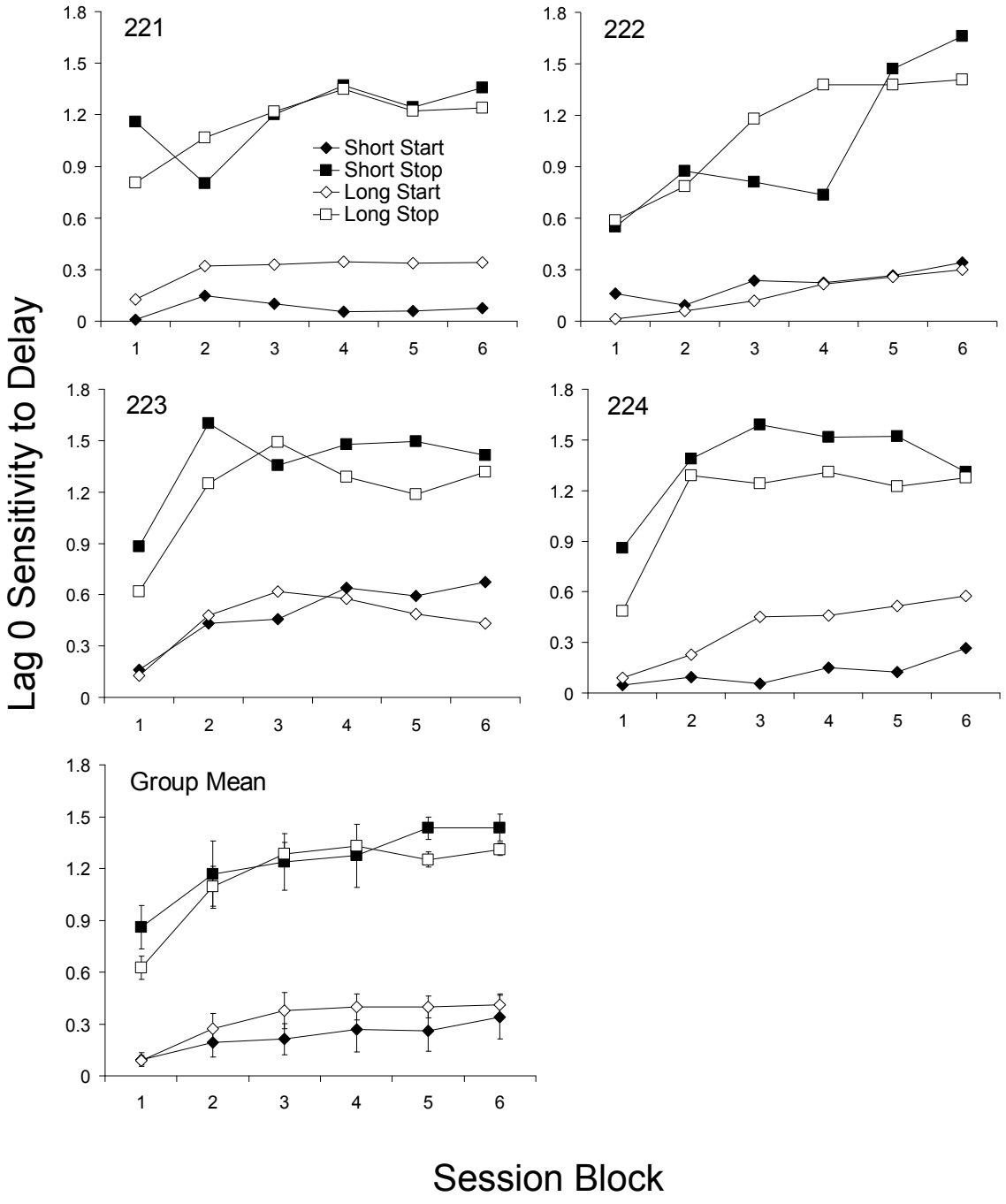

Figure 4.4. Sensitivity of start and stop times to scheduled terminal-link delay on food trials for each block of 12 initial- and terminal-link cycles, for individual-subject and group mean data. Diamonds and squares represent start and stop time sensitivity coefficients, respectively. Solid data points are coefficients obtained from the Short condition. Open data points are coefficients obtained from the Long condition. Bars show standard error.

Figure 4.4 shows start and stop time sensitivity (i.e., the slope of the regression line) to terminal-link delay as a function of session block for individual subjects and the group mean. We entered individual start and stop time data from Figure 4.4 into repeated-measures ANOVAs with condition (Short or Long) and block as factors. Across subjects, start time sensitivity to delay was only affected by session block, that is, there was a main effect of block, $F(5,15) = 8.33$, $p<0.001$, but not of condition, $F(1,3) = 1.29$, *ns*, and no block by condition interaction, $F(5,15) = 1.41$, *ns*. Results for stop times were similar: there was a main effect of block, $F(5,15) = 7.41$, $p<0.01$, but the main effect of condition, $F(1,3) = 1.32$, *ns*, and the block by condition interaction, $F(5,15) = 1.14$, *ns* were not significant. Post-hoc analyses (Tukey's HSD) indicated that start and stop time sensitivities ceased changing systematically after the second block. These multiple regression results indicate there was no systematic difference in sensitivity to delay in Short versus Long conditions, which is consistent with timescale invariance for start and stop times.

Figure 4.5 shows start and stop times as a function of terminal-link delay for individual subjects. Each data point represents the mean of the start or stop times from left or right no-food trials from blocks 4-6 because previous studies (Kyonka & Grace, 2007, 2009) have reported that temporal control in rapid acquisition procedures stabilizes at or before the halfway point in sessions of 72 cycles' duration and multiple regression analyses showed that start and stop time sensitivity to delay does not change systematically over those Blocks. There are approximately four hundred data points per panel, representing mean start and stop times from the second half of a single session for left and right alternatives, for each session of conditions 1 and 2.

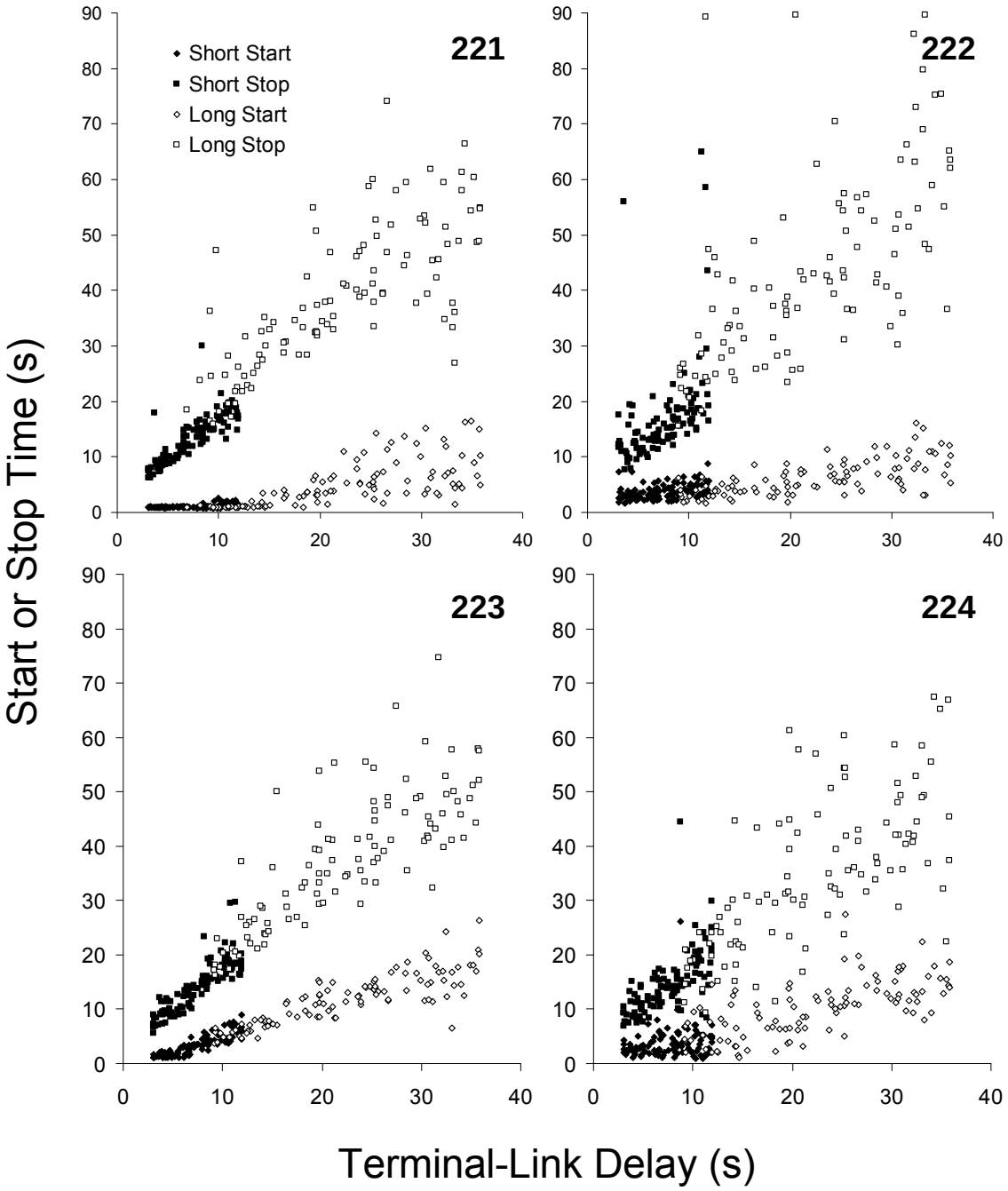

Figure 4.5. Start times (diamonds) and stop times (squares) as a function of terminal-link delay to reinforcement, for all subjects. Each data point represents the mean start or stop time for a single session and terminal link alternative (left or right). Solid and open data points were obtained from Short and Long conditions, respectively.

For all subjects, start and stop times increased linearly and tended to become more variable as a function of delay. Averaged across terminal links and subjects, the slope and intercept, respectively, obtained from regressions of mean start times on terminal-link delay were 0.26 (*S.E.* = 0.13) and 0.90 (*S.E.* = 0.77) for the Short condition and 0.38 (*S.E.* = 0.05) and -0.31 (*S.E.* = 0.77) for the Long condition, with VACs of 0.32 (*S.E.* = 0.16) and 0.53 (*S.E.* = 0.06), respectively. Averaged slopes and intercepts obtained from regressions of mean stop times on terminal-link delay were 1.51 (*S.E.* = 0.13) and 3.54 (*S.E.* = 0.25) for the Short condition and 1.27 (*S.E.* = 0.05) and 10.71 (*S.E.* = 0.86) for the Long condition, with VACs of 0.56 (*S.E.* = 0.09) and 0.59 (*S.E.* = 0.01), respectively.

Across subjects, there was no systematic difference in temporal control in the Short and Long conditions. However, start time sensitivity was consistently greater in Long than in Short sessions for Pigeons 221 and 224. The greater sensitivity for two subjects could imply greater relative precision in timing longer delays, a floor effect or influence of nontemporal factors on start (but not stop) times.

Table 4.2. Pearson correlations (*r*) between coefficients of variation calculated from left or right start or stop times from blocks 4-6 of individual sessions.
*** p < 0.05, ** p < 0.01, *** p < 0.001**

	Start		Stop	
Pigeon	Short	Long	Short	Long
221	0.53***	0.33**	-0.03	-0.02
222	0.23**	0.22*	-0.1	-0.15
223	0.1	0.11	0.01	0.19
224	0.31**	-0.27**	0.02	0.01
Group Mean	0.29	0.1	-0.03	0.01
S.E.	0.09	0.13	0.03	0.07

The scalar property is a fundamental principle of theories of timing (Gibbon, 1977; Gallistel & Gibbon, 2000; Killeen & Fetterman, 1988) asserting that variability in responding should increase in proportion to schedule value such relative variability is

constant. The visual impression given by Figure 4.5 is that the scalar property applied to start and stop times in this experiment. To investigate this possibility, we calculated coefficients of variation (CVs) for start and stop times of individual subjects across the last thee Blocks, when behavior had stabilized using the method of Kyonka and Grace (2007). Averaged across subjects, start time CVs were 0.30 (*S.E.* = 0.05) and 0.40 (*S.E.* = 0.04) in Short and Long conditions, respectively. Stop time CVs were 0.14 (*S.E.* = 0.01) and 0.15 (*S.E.* = 0.01) in Short and Long conditions, respectively. We entered mean coefficients of variation from individual subjects into a repeated-measures ANOVA with measure of temporal control (start or stop time) and condition (Short or Long) as factors. Stop times were significantly more precise than start times, $F(1,3) = 32.08$, $p<0.05$, but there was no effect of condition and no measure by condition interaction, $F(1,3) = 3.97$ and $F(1,3) = 3.38$, respectively, both *ns*. These results support the visual impression that there is no systematic difference in temporal control across conditions. The scalar property predicts no effect of condition and zero correlation between CV and delay. Table 4.2 shows Pearson correlations of start and stop time CVs from Short and Long conditions, with terminal-link delay for individual subjects. For stop times, correlations were small, nonsignificant and not systematically positive or negative across subjects. By contrast, relative variability in start times increased as a function of delay for 3 out of 4 subjects when delays were Short and 2 out of 4 when delays were Long.

4.2.5 Discussion

The goals of this experiment were to characterize how response allocation and temporal control of terminal-link responding in a rapid acquisition concurrent-chains procedure was affected by terminal-link duration. We asked whether the relationship

between response allocation and immediacy ratio was nonlinear, whether absolute terminal-link duration affected it, and whether there was any effect on temporal control.

Response allocation data were consistent with results of other studies that manipulated absolute terminal-link duration and with those in which terminal-link delays changed unpredictably across sessions. Relative immediacy ratios from the current, but not previous sessions determined response allocation was determined and sensitivity to current-session immediacy ratio was greater in long sessions in which delays summed to 45 s than in short sessions in which delays summed to 15 s. In contrast to predictions generated by the generalized matching law but consistent with results of other rapid acquisition experiments, log response ratios were not a linear function of log immediacy ratios, but rather fell in two clusters determined by whether the left or right alternative was preferred.

Start and stop times, measures of temporal control, adapted to delays and stabilized within sessions. Stop times from the second half of sessions were timescale-invariant; that is, the relationship between stop time and schedule value was homogeneous across the full range of terminal-link delays. Moreover, the scalar property applied to stop times: their relative variability was constant. By contrast, relationships between start times and terminal-link delays were less consistent across subjects. There was no systematic difference in sensitivity to delay for Pigeons 222 or 223, but sensitivity was consistently lower in the Short than the Long condition for Pigeons 221 and 224 (even though they experienced conditions in a different order).

Although there was considerable variability across subjects, the scalar property did not always apply to start times. Relative variability increased as a function of terminal-link delay for all subjects (significantly for all subjects except Pigeon 223) in the Short

condition. In the Long condition, relative variability increased significantly for Pigeons 221 and 222, increased (but not significantly) for Pigeon 223 and significantly decreased for Pigeon 224. Kyonka and Grace (2007) noted that coefficients of variation were higher for start than for stop times in their experiment. They hypothesized that the stimulus change accompanying terminal-link onset may have an excitatory effect on responding early in terminal links, and that or other non-temporal factors may influence start times. Further, the heterogeneity of start time results across subjects in the present experiment suggests they may be a less reliable measure of temporal control than stop times in concurrent chains procedures.

In addition to the convergent replications of the terminal-link effect of greater sensitivity to longer absolute terminal links, the scalar property and timescale invariance of stop times, and rapid adaptation of choice and timing to novel unpredictably-changing terminal-link delays, the present experiment offers insights into the functional relationships between choice, relative immediacy and absolute terminal-link duration. Using model comparison, we characterized the effect of absolute terminal-link duration on the clustering of response ratios observed in concurrent-chains experiments featuring unpredictably changing delays. Within-cluster sensitivity to relative immediacy was unaffected by terminal-link duration, however the magnitude of the difference between clusters was greater when terminal links were long than when they were short.

In this experiment, subjects experienced three conditions in which terminal link delays were always short or always long, and one in which pairs of delays in a pseudorandom half of sessions were short and the remainder were long. Initial-link response allocation in short sessions of this Mixed condition tended to be less sensitive to

relative immediacy than in the Short condition. Similarly, sensitivity was lower in long sessions of the Mixed condition than in the Long condition. Although there was no effect of log immediacy ratios from previous sessions on initial-link responding, greater variability in delays across sessions (but within a condition) apparently decreased sensitivity to immediacy.

One goal of this experiment was to investigate whether it was possible to identify the mechanism that produces the terminal-link effect using a rapid acquisition concurrent-chains procedure. Two possible mechanisms generated two different predictions. If individuals were more sensitive to immediacy ratios when delays were overall longer because ratios were easier to discriminate, acquisition of preference should have been faster in long than in short sessions. There was no evidence that rate of acquisition depended on the absolute duration of terminal links. If the difference in sensitivity was attributable to a differential value of relative immediacies, as DRT assumes, the difference in sensitivity should apply within-cluster rather than between clusters. In other words, when the piecewise linear model was applied to initial-link performance, the slope for long-session data should have been greater than the slope fitted to data from short sessions. Parameter estimates from the piecewise linear model with six parameters show this was the case for 3 out of 4 subjects, all those that showed a terminal-link effect. However, across subjects, separate slopes for short and long sessions did not improve VAC over a piecewise linear model that assumed the same slope for short and long sessions. The present data do not confirm either hypothesis, however, initial-link performance is consistent a differential valuation of relative immediacy for short and long terminal-link delays.

5 Chapter 5 Rapid acquisition of choice: Effects of unpredictable changes to multiple dimensions

5.1 Notes on Experiments 4 and 5

Chapter 5 begins with Experiments 4 and 5, published as "Rapid Acquisition of Preference in Concurrent Chains When Alternatives Differ on Multiple Dimensions of Reinforcement" in the *Journal of the Experimental Analysis of Behavior, 89*, 2008. Citations of Kyonka and Grace (2008) in Chapters 3 and 4 refer to this article. Four pigeons responded in a minimal-variation concurrent chains procedure in which multiple dimensions were changed across sessions according to independent, random binary sequences. There have been investigations of the combined effects of immediacy and magnitude or other dimensions (notably, the body of research on self control, see Logue, 1988 for a review) on responding. However, Experiments 4 and 5 are unusual in that they involved simultaneous, independent, parametric manipulations of multiple reinforcer dimension ratios. In changing reinforcer contingencies unpredictably across sessions, Experiments 4-6 are unique.

Experiments 4-6 are unlike Experiments 1-3 in that measures of temporal control were not collected. This chapter represents a different approach to characterizing the process that generates choice: does responding reflect control by multiple dimensions, and if so, what is the nature of that control? Are within-session adjustments to preference in Experiments 4 and 5 as abrupt as adjustments in Experiment 1? If so, when did they tend to occur and would it be possible to predict pigeons' final, stable level of preference as a function of immediacy, magnitude and (in Experiment 5) probability ratios?

5.2 Rapid Acquisition of Preference in Concurrent Chains When Alternatives Differ on Multiple Dimensions of Reinforcement

5.2.1 Abstract

Pigeons responded in a concurrent-chains procedure in which terminal-link reinforcer variables were changed unpredictably across sessions. In Experiment 1, the terminal-link schedules were fixed-interval (FI) 8 s and FI 16 s, and the reinforcer magnitudes were 2 s and 4 s. In Experiment 2 the probability of reinforcement (100% or 50%) was varied with immediacy and magnitude. Multiple-regression analyses showed that pigeons' initial-link response allocation was determined by current-session reinforcer variables, similar to previous studies which have varied only immediacy (Grace, Bragason, & McLean, 2003). Sensitivity coefficients were positive and statistically significant for all reinforcer variables in both experiments. Analyses of responding within individual sessions showed that final levels of preference for "dominated" sessions, in which all reinforcer variables favored the same terminal link, were more extreme than for "tradeoff" sessions in which at least one reinforcer variable favored each alternative. This result implies that response allocation was determined by multiple reinforcer variables within individual sessions, consistent with the concatenated matching law. However, in Experiment 2, there was a nonlinear (sigmoidal) relationship between response allocation and relative value,

which suggests the possibility that reinforcer variables may interact during acquisition, contrary to the matching law.

Keywords: acquisition, concurrent chains, reinforcer immediacy, reinforcer magnitude, reinforcer probability, key peck, pigeons

5.2.2 Introduction

More than four decades of research on behavioral choice has shown that response allocation in concurrent schedules and concurrent chains schedules is determined by characteristics of the reinforcing stimuli that are contingent on responding, including their rate (Herrnstein, 1961), magnitude (Catania, 1963), immediacy (the reciprocal of delay to reinforcement; Chung & Herrnstein 1967), hedonic quality (Killeen, Cate, & Tran, 1993; Miller, 1976), and probability (Schneider, 1968; Spetch & Dunn, 1987). Most studies have manipulated one reinforcer dimension parametrically and observed the resulting changes in steady-state response allocation, with Herrnstein's (1961) demonstration that response allocation was approximately equal to relative reinforcer rate in concurrent schedules – the matching law – being the best-known result. But what happens when the choice alternatives differ on more than one dimension? Baum and Rachlin (1969, p. 870) proposed that organisms allocate time (and, by extension, responses) between two alternatives according to the relative value obtained, such that "the ratio of times allocated [between two activities] equals the ratio of the values of the activities." They suggested that the simplest possible way of determining value was a multiplicative combination of the relative reinforcer dimensions:

$$\frac{B_L}{B_R} = \prod_{i=1}^{n} \frac{X_{iL}}{X_{iR}} = \frac{V_L}{V_R}.$$

Equation 5.1

In Equation 5.1, subscripts *L* and *R* refer to left and right alternatives, *B* is response rate, *V* is value, and X_{iL} and X_{iR} are the values of *i*th reinforcer dimension X_i associated with left and right alternatives, respectively. According to Baum and Rachlin's proposal, the matching law applies to the relative value obtained from the alternatives, which is determined by a multiplicative concatenation of reinforcer ratios.

Rachlin (1971) explored the conceptual implications of Baum and Rachlin's (1969) extension of the matching law. Rachlin (1971) noted that the equality of response allocation and relative value was an assumption rather than a testable hypothesis and thus the utility of the matching law was that it "circumscribe[d] [the] search for reinforcers in any situation" (p. 251). Killeen (1972) proposed that in its most general form, the theoretical matching law should allow arbitrary transformations of each reinforcer dimension on each alternative. The transformation that has been most useful to date is a power function, which implies that the effects of different reinforcer dimensions are linearly independent in logarithmic terms:

$$\log\left(\frac{B_L}{B_R}\right) = \left[\sum_{i=1}^{n} a_i \log\left(\frac{X_{iL}}{X_{iR}}\right)\right] + \log b.$$

Equation 5.2

Equation 5.2 is known as the concatenated generalized matching law (Davison, 1983; Davison & McCarthy, 1988), and is equivalent to the log transformation of Equation 5.1 except that sensitivities (a_i) to each reinforcer dimension need not be equal to 1, and *b* represents a constant preference for responding to either alternative that is independent of

any reinforcer ratio (i.e., bias). Equation 5.2 is a testable empirical model for choice between two alternatives because it requires that different reinforcer dimensions have additive and independent effects on response allocation.

One application of Equation 5.2 has been to the study of self-control which refers to choice situations in which alternatives differ in terms of immediacy and magnitude. When offered a choice between a larger, more delayed reinforcer and a smaller, more immediate one, self-control is defined as preference for the former and impulsivity as preference for the latter. Self-control increases as the minimum delay between the choice and reinforcement increases (see Logue, 1998, for a review), consistent with the assumption that the effects of relative immediacy and magnitude on preference are additive and independent, as has been confirmed by a number of studies (Grace 1995; Grace, Bedell, & Nevin, 2002; Logue, Forzano & Tobin, 1992; Rodriguez & Logue, 1986).

Studies in which other pairs of reinforcer dimensions have been manipulated parametrically have generally supported the additivity and independence assumptions of Equation 5.2. The earliest work supporting a "multiplicative power law" (Hamblin & Miller, 1977; Schneider, 1973; Todorov, 1973) employed concurrent variable-interval (VI) VI (conc VI VI) schedules that varied in reinforcer rate, magnitude, or both. More recently, McLean and Blampied (2001) have confirmed the independence of rate and magnitude in concurrent schedules. They also reported no effect of absolute rate or magnitude on response allocation, which is consistent with Equation 5.2 but contrary to other research studying the effects of different rates and magnitudes of reinforcers (cf. Alsop & Elliffe, 1988; Davison, 1988; Logue & Chavarro, 1987). Mazur (1988) used an adjusting-delay procedure to determine fixed-delay indifference points for a series of standard schedules

that varied in terms of reinforcer magnitude and probability. He found that indifference points increased with the "total reinforcer access ratio" (Neuringer, 1967) – the multiplicative combination of magnitude and probability ratios. Leon and Gallistel (1998) found that rats' time allocation in conc VI VI schedules matched a multiplicative combination of rate and electrical frequency of brain stimulation. Berg and Grace (2004) showed that relative rates of conditioned reinforcement and terminal-link immediacy had additive and independent effects on response allocation in concurrent chains.

Overall, the concatenated matching law provides a good quantitative framework for describing steady-state choice when outcomes can differ in terms of one or more reinforcer dimensions. However, no studies have tested whether the concatenated matching law is valid for choice in transition. Knowing whether the same principles apply to choice under dynamic conditions as well as steady state is essential for developing more comprehensive models of choice behavior.

One approach to studying choice in transition involves presenting subjects with alternatives that change unpredictably across sessions. For example, Schofield and Davison (1997) trained pigeons on conc VI VI schedules in which scheduled reinforcer ratios were either of two reciprocal values (1:2 and 2:1, 1:4 and 4:1, or 1:8 and 8:1, depending on condition) and the location of the richer alternative was determined each session by a 31-step pseudorandom binary sequence. To assess the degree of control exerted by the reinforcer ratio from the current and previous sessions on choice, Schofield and Davison (1997) regressed log response and time allocation ratios on obtained log reinforcer ratios from the current session (Lag 0) and nine prior sessions (Lags 1 through 9). They found that after pigeons had been exposed to three presentations of the pseudorandom sequence

(i.e., 93 sessions), sensitivity was greatest for Lag 0 ratios and close to zero for all others, suggesting performance was determined by reinforcer ratios in the current session. Grace, Bragason, and McLean (2003) obtained similar results in concurrent chains. In their Experiment 1, the left terminal-link schedule was always fixed-interval (FI) 8 s and the right terminal link was either FI 4 s or FI 16 s, according a 31-step pseudorandom binary series. Multiple-regression analyses showed that after three presentations of the pseudorandom sequence, Lag 0 sensitivity to immediacy coefficients were positive and significant for all subjects, whereas higher-lag coefficients were near zero. Subsequent research confirmed that pigeons' response allocation can track unpredictable changes in immediacy when a different pair of terminal-link schedules was presented in each session (Grace & McLean, 2006).

What might determine response allocation if multiple reinforcer dimensions, for example, immediacy and magnitude, change unpredictably across sessions? One possibility is that the concatenated matching law applies to choice in transition. According to this view, immediacy and magnitude ratios should combine additively and independently to determine reinforcer value, with acquisition of preference during any temporal epoch (e.g., an individual session) resulting from differences in relative value. Another possibility is that only one dimension determines response allocation in a given epoch, but the controlling dimension changes across epochs. For example, subjects' preference might be determined by immediacy in some sessions but magnitude in others. In that case, analysis of data at the molar level (i.e., aggregated across sessions) still might reveal control by both reinforcer dimensions, although the underlying processes would be different.

We describe two experiments in which pigeons responded in a concurrent-chains procedure where the terminal links differed on multiple reinforcer dimensions that changed unpredictably across sessions. In Experiment 4, the terminal links were always FI 8 s and FI 16 s and the reinforcer magnitudes were 4-s and 2-s access to grain. For each session, the assignment of FI schedules and reinforcer magnitudes to the left and right terminal links was determined according to independent random binary sequences. In Experiment 5, reinforcer probability (100% or 50%) was changed across sessions, along with immediacy and magnitude, according to independent random sequences. For both experiments, we planned to determine whether the molar data supported the additivity and independence assumptions of the concatenated matching law. Assuming that the molar data showed control by multiple reinforcer dimensions, we then sought to determine whether responding over more limited temporal epochs was controlled by multiple or single dimensions.

To distinguish between these possibilities, we planned to examine performance during individual sessions and cycles. The critical comparison was between sessions in which both dimensions favored one alternative (described as "dominated" because the same alternative was the superior on all dimensions), and those in which at least one dimension favored each alternative ("tradeoff" sessions). If responding is controlled by multiple dimensions, as the concatenated matching law predicts, preference in dominated sessions should be more extreme than preference in tradeoff sessions. However, if only one dimension controlled responding at a time, there should be no systematic difference between preference in dominated and tradeoff sessions.

5.3 Experiment 4: immediacy and magnitude

5.3.1 Method

5.3.1.1 Subjects

Four pigeons of mixed breed and sex, numbered 191, 192, 193 and 194, were maintained at 85% ad libitum weight plus or minus 15g through appropriate post-session feedings of a grain, chickpea, and cod liver oil mixture. Pigeons were housed individually in a vivarium with a 12-hr:12-hr light:dark cycle (lights on at 7 a.m.) with free access to water and grit. All had previous experience with a variety of choice procedures including concurrent chains, but had not previously served in experiments in which terminal-link contingencies changed unpredictably across sessions.

5.3.1.2 Apparatus

Four standard three-key operant chambers were used, measuring 32 cm deep x 34 cm wide x 34 cm high. The keys were 21 cm above the floor and arranged in a row 10 cm apart. In each chamber there was a houselight located above the center key and a grain magazine with a 5 cm x 5.5 cm aperture that was centered 6 cm above the floor. The houselight provided general illumination at all times except during reinforcer delivery. The magazine contained wheat and was illuminated during reinforcement. A force of approximately 0.15 N was necessary to operate each key. Each chamber was enclosed in a sound-attenuating box, and ventilation and white noise were provided by an attached fan. Experimental events were controlled through a microcomputer and MED-PC® interface located in an adjacent room.

5.3.1.3 Procedure

Because subjects were experienced, training began immediately in the first condition. With few exceptions, sessions were conducted daily at approximately the same time (11 a.m.).

Sessions ended after 72 initial- and terminal-link cycles (trials) had been completed or 70 minutes, whichever came first. At the start of each trial, the side keys were lighted white to signal the initial links. A terminal-link entry was assigned pseudorandomly to the left or right key, with the constraint that in every block of 12 trials, 6 entries were assigned to each key. An initial-link response produced a terminal-link entry if: (a) it was made to the pre-selected key; (b) an interval selected from a VI 10-s schedule had timed out; and (c) a 1-s changeover delay was satisfied, i.e., at least 1 s had elapsed after a changeover to the side to which terminal-link entry was arranged.

The VI 10-s initial-link schedule did not begin until the pigeon first pecked either key. In this way, pausing after completion of the terminal links was excluded from initial-link time. The VI 10-s schedule contained 12 intervals constructed from an exponential progression (Fleshler & Hoffman, 1962). Separate lists were sampled without replacement so that all 12 intervals would be used three times each for the left and right terminal links every session.

Terminal-link entry was signaled by changing the color of the side key that produced the terminal link (left key to red, right key to green for two subjects and vice versa for the other two) and darkening the other side key. Terminal-link responding was reinforced with access to grain according to FI schedules. The FI schedule values were always 8 and 16 s. The reinforcer magnitudes were always 2 s and 4 s. Across sessions,

the locations of the FI schedules and reinforcer magnitudes were changed according to independent random binary sequences. These sequences were determined by downloading two series of random digits generated[1] by a radioactive decay process (retrieved on July 14, 2003 from www.fourmilab.ch/hotbits/).

There were four possible configurations of sessions: Both shorter delay (FI 8 s) and larger magnitude (4 s) assigned to the left; both shorter delay and larger magnitude assigned to the right; shorter delay assigned to the left and larger magnitude to the right; and shorter delay assigned to the right, larger magnitude to the left. Thus, a random half of the sessions were "dominated," in that both the immediacy and magnitude favored the same side, whereas the other half were "tradeoff" sessions in which the shorter delay was associated with one alternative and the larger magnitude with the other. In tradeoff sessions, the expected income, or reinforcer access, for each terminal link, calculated as seconds of access to food per terminal-link delay, was the same for both alternatives; that is, 2-s reinforcer magnitude after an 8-s delay and 4-s after a 16-s delay both provided 0.25 s reinforcer access per 1 s of terminal-link delay.

Experiment 4 lasted 93 sessions. Based on visual inspection of plots of sensitivity to immediacy and magnitude across sessions (10- and 20-session equally-weighted moving averages) for each subject, the data included in these analyses are from the last 45 sessions. Owing to an equipment problem, Pigeon 191 failed to complete 28 nonconsecutive sessions, but none of these were during the last 45 sessions. Pigeon 194 did not respond in three of the final 45 sessions, and consequently those sessions were replaced with the 46th-, 47th- and 48th-to-last sessions.

5.3.2 Results

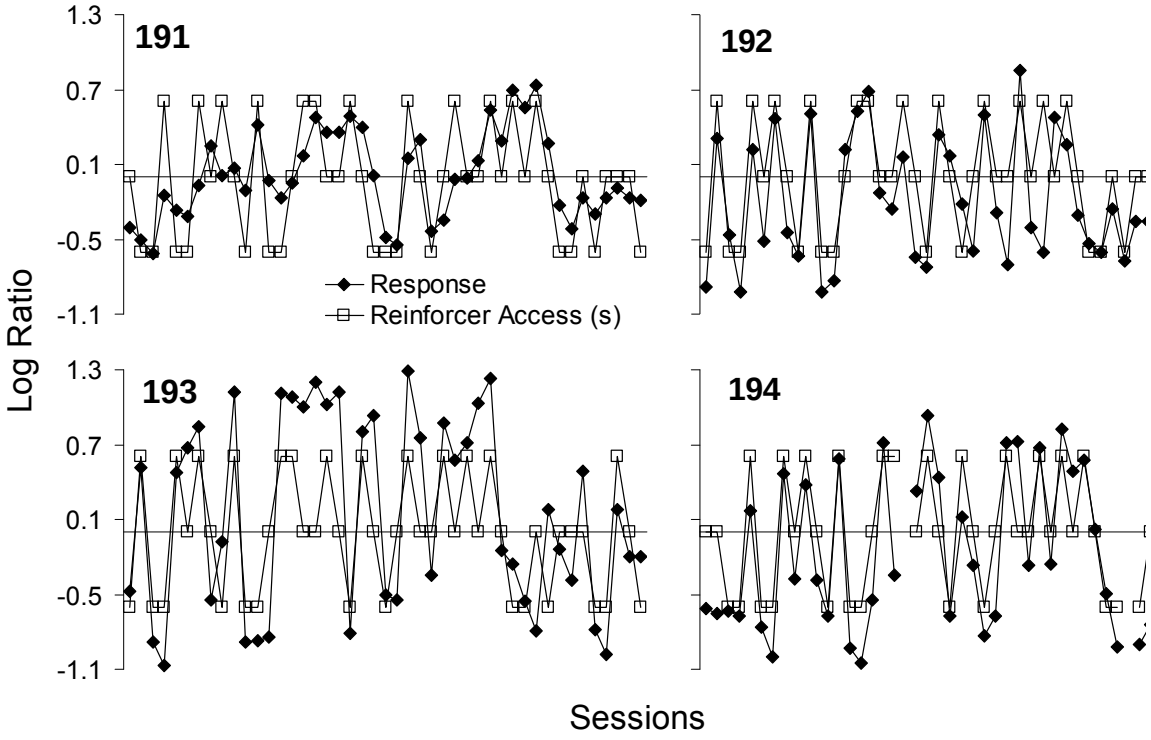

Figure 5.1. Log initial-link response ratios and log terminal-link reinforcer access ratios across the last 45 sessions of Experiment 4. Reinforcer access ratios were calculated as described in the text.

Figure 5.1 shows the log initial-link response and log terminal-link reinforcer access ratios (left key to right key) for all subjects across the last 45 sessions of Experiment 1. Reinforcer access is the product of reinforcer magnitude and immediacy, and represents duration of access to grain relative to time spent waiting in the terminal link. The log terminal-link reinforcer access ratio is thus computed as $\log((M_L/D_L)/(M_R/D_R))$, where *M* is magnitude, *D*, delay, and subscripts *L* and *R* refer to left and right alternatives, respectively. Log terminal-link reinforcer access ratios were 0.6, 0, and -0.6 for left-dominated, tradeoff, and right-dominated sessions, respectively. Each data point represents performance in a single session. For three of four pigeons, response allocation tracked the changes in the reinforcer access ratio: Preference for the left alternative generally increased from one session to the next if relative reinforcer access on the left alternative increased, regardless of

whether from right-dominated to tradeoff, right-dominated to left-dominated, or tradeoff to left-dominated. Conversely, if the reinforcer access ratio decreased, preference shifted toward the right alternative. The exception was response allocation for Pigeon 193, which generally favored the alternative with the shorter delay to reinforcement.

To assess quantitatively the degree of control over response allocation by the immediacy and magnitude ratios from current and prior sessions, we applied a generalized-matching model to the data:

$$\log\frac{B_{0L}}{B_{0R}} = a_{0D}\log\frac{1/D_{0L}}{1/D_{0R}} + a_{0M}\log\frac{M_{0L}}{M_{0R}} + a_{1D}\log\frac{1/D_{1L}}{1/D_{1R}} + a_{1M}\log\frac{M_{1L}}{M_{1R}} + a_{2D}\log\frac{1/D_{2L}}{1/D_{2R}} + a_{2M}\log\frac{M_{2L}}{M_{2R}} + \log b$$

Equation 5.3

In Equation 5.3, B is initial-link responding, D is terminal-link delay to reinforcement, M is reinforcer magnitude, a's are sensitivity coefficients, and log b is response bias. The subscripts L and R refer to the left and right alternative, respectively, and numeric subscripts refer to session lag. To limit the complexity of the model, only terms up to Lag 2 were included because previous research generally has found no evidence of significant control by higher lags (Grace, Bragason, & McLean, 2003; Grace & McLean, 2006; Schofield & Davison, 1997).

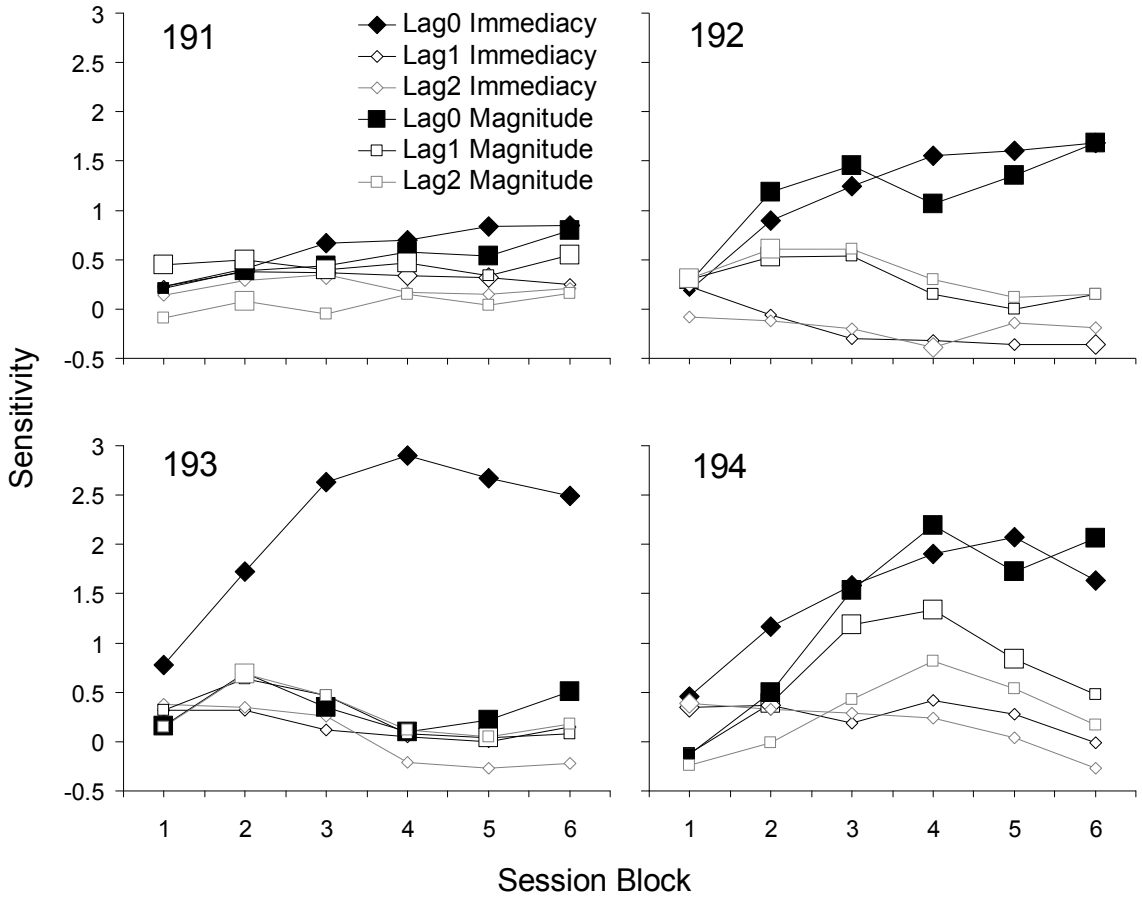

Figure 5.2. Sensitivity to log immediacy and magnitude ratios for Lag 0 through Lag 2 for each block of 12 trials. Diamond data points represent immediacy ratios and square data points represent magnitude ratios. Larger data points represent statistically significant coefficients.

Multiple regressions were used to obtain parameter estimates for Equation 5.3 for each block of 12 trials within each session for individual subjects. Figure 5.2 shows the resulting sensitivity coefficients for immediacy and magnitude ratios from the current session (Lag 0) and two previous sessions (Lags 1 and 2). Larger data points indicate coefficients corresponding to terms in the regression that were significantly greater or less than zero ($p < .05$). For all subjects, Lag 0 sensitivity coefficients for both immediacy and magnitude increased over the course of the session, whereas those for higher lags either did not change systematically or decreased. Lag 0 sensitivity to immediacy was always greater than Lag 0 sensitivity to magnitude for Pigeon 193, but there was little difference for the

other subjects. For the second half of the session, average Lag 0 sensitivities to immediacy and magnitude were 1.74 [SD = 0.78] and 0.96 [SD = 0.43], respectively.

An important assumption of the concatenated matching law is that the effects of different reinforcer dimensions on log response allocation are additive and independent. Specifically, there should be no interaction between immediacy and magnitude. To test this prediction, we analyzed data from the second half of each session with a model that only included Lag 0 coefficients:

$$\log\frac{B_{0L}}{B_{0R}} = a_{0D}\log\frac{1/D_{0L}}{1/D_{0R}} + a_{0M}\log\frac{M_{0L}}{M_{0R}} + \log b$$

Equation 5.4

We first used multiple regression to obtain parameter estimates of Equation 5.4 for data from the final three blocks of each session (second session half) and then computed the incremental variance accounted for by the addition of the interaction term (i.e., the product of log immediacy and magnitude ratios in the current session). The results, including specific values for a_{0D}, a_{0M}, and log b (the intercept), are shown in Table 5.1. Equation 5.4 described the data reasonably well, accounting for an average of 75% of the variance across subjects. The interaction term was significant for Pigeon 192 (B = 1.36, R^2_{inc} = .03, $p < .05$) but not for the remaining subjects.

Table 5.1. Results of regressing log response allocation (averaged over the final three blocks of each session) on Lag 0 log immediacy and magnitude ratios from Experiment 1. Sensitivity coefficients for immediacy and magnitude are the unstandardized weights obtained prior to the addition of the interaction term. R^2_{inc} refers to the increment in R^2 when the interaction value was applied. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

Pigeon	Immediacy	Magnitude	Intercept	R^2	Interaction	R^2_{inc}
191	0.92***	0.46***	0.00	0.62	-0.66	0.02
192	1.52***	1.03***	-0.08	0.79	1.36*	0.03
193	2.78***	0.34	0.22	0.79	-0.48	0.00
194	1.93***	1.27***	-0.14	0.81	0.66	0.00

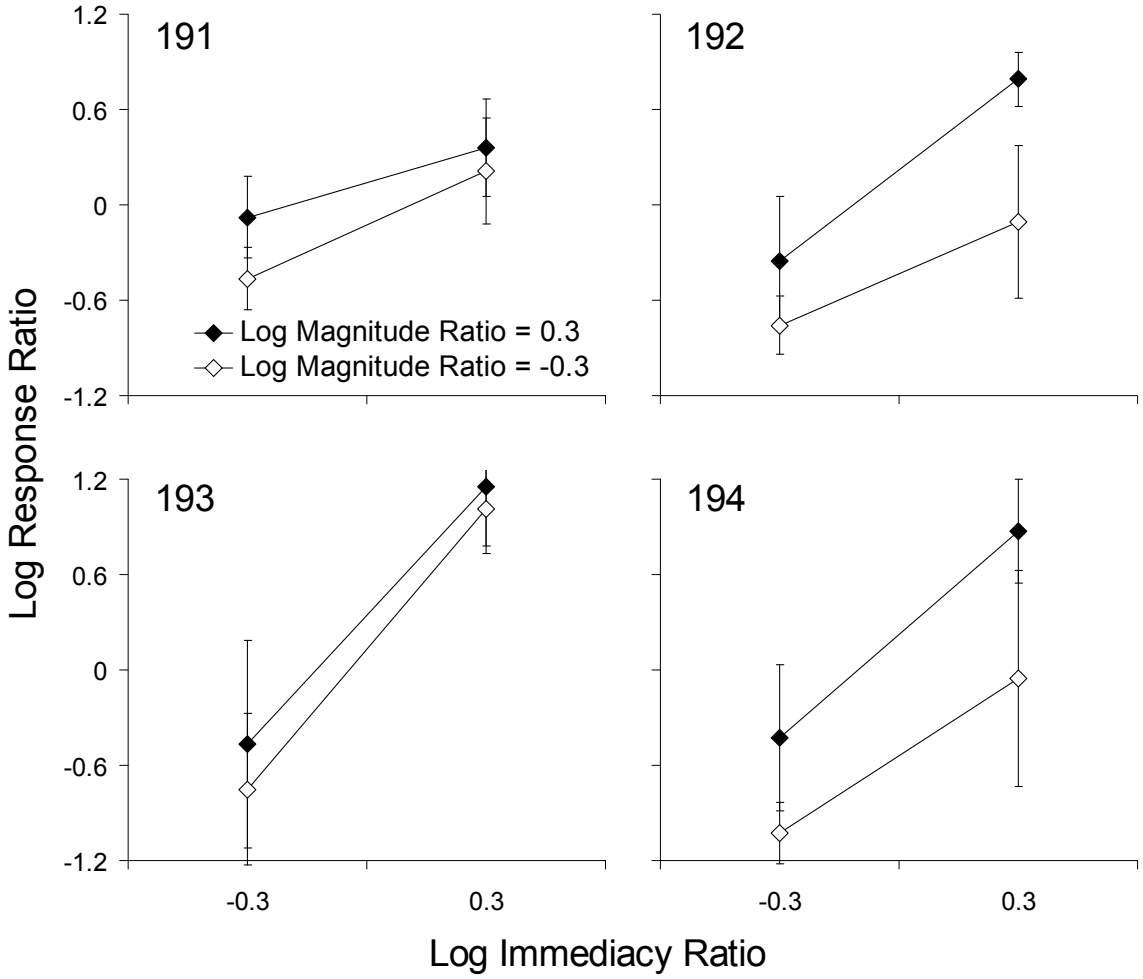

Figure 5.3. Mean log response ratio as a function of log immediacy ratio when log magnitude ratio was positive (filled data points) and negative (unfilled). Error bars represent the standard deviation.

To display the interaction graphically, Figure 5.3 plots the average log response ratio from the second half of each session for each configuration of immediacy and magnitude ratios for individual subjects. For Pigeons 192 and 194, the relative effect of magnitude was greater when immediacy favored the left alternative than when it favored the right. But the opposite result was obtained for Pigeons 191 and 193: The relative effect of magnitude was greater when immediacy favored the right than when it favored the left. Thus across subjects there was no evidence for a consistent interaction between immediacy and magnitude.

Overall, the results in Figures 5.2 and 5.3 and Table 5.1 suggest that effects of relative immediacy and magnitude on response allocation were additive and independent, and that control by both variables increased over the course of the session. However, those results were based on data aggregated across sessions, and so the conclusions may not be representative of individual sessions. If a single reinforcer dimension selected at the beginning of each session determined response allocation, there would be no systematic difference between the final level of preference reached in tradeoff and dominated sessions because the relative values should be equal. By contrast, concatenated generalized matching requires that both reinforcer dimensions determine response allocation in individual sessions, and therefore response allocation should be less extreme in tradeoff compared to dominated sessions.

To investigate response allocation within sessions, a reliable measure of pigeons' final, stable response allocation in individual sessions was necessary. To achieve this, we first examined the nature of within-session changes in preference by plotting, for all subjects and sessions, the cumulative number of responses to the right initial link as a function of the cumulative number of responses to the left initial link for each of the 72 trials (cf. Gallistel, Mark, King, & Latham, 2001). The resulting cumulative-response scatterplots could be described in terms of a few general categories. Figure 5.4 shows representative plots for one subject (Pigeon 194). At the start of the session, response allocation was usually indifferent (see panels A and D) or favored one alternative (panels B and C). Changes in strength of preference could occur at any point in the session and tended to be abrupt rather than gradual (see panel C, for example). Sometimes response allocation changed multiple times within the session (see panel D).

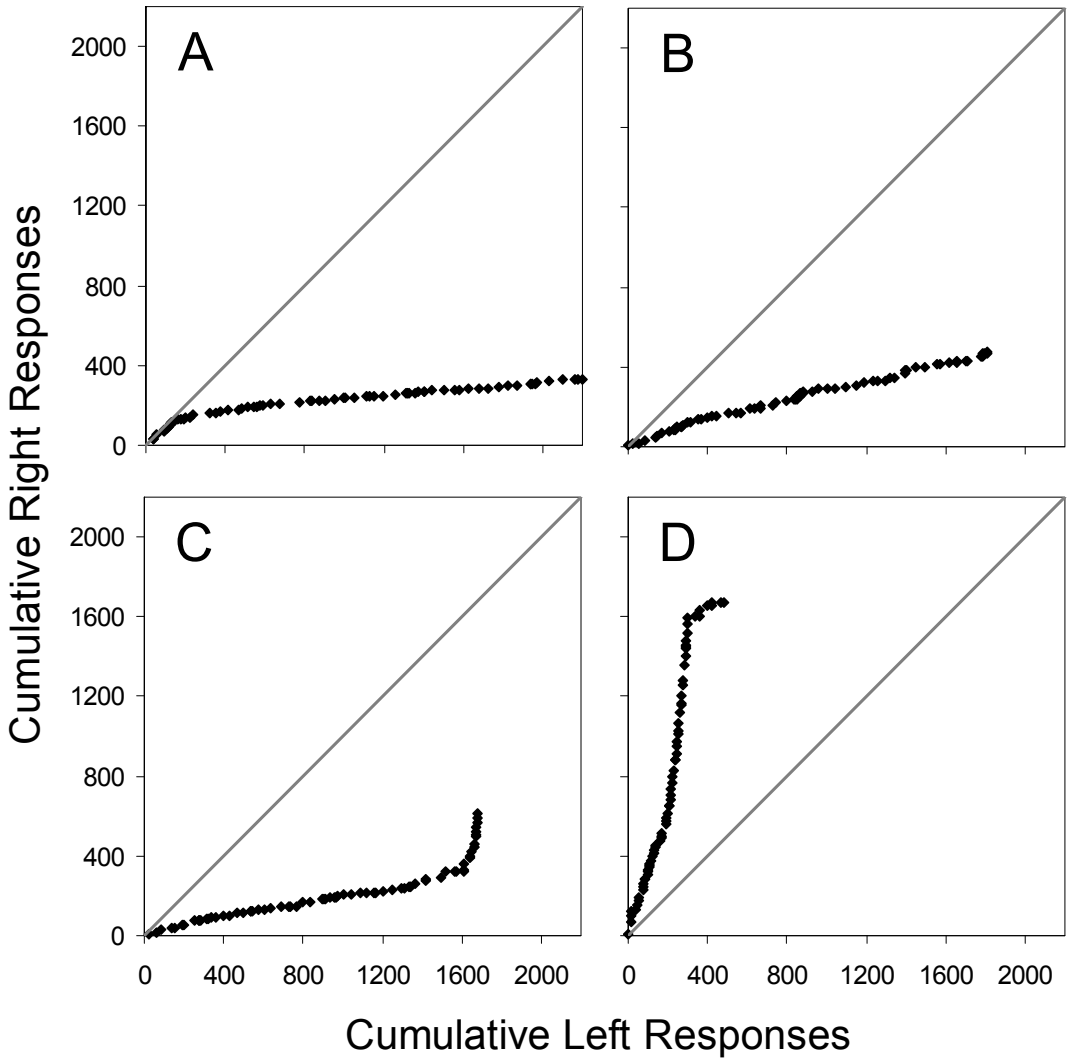

Figure 5.4. Each panel plots the cumulative number of responses made to the right initial link (*y*) against the cumulative number of responses made to the left initial link (*x*) during one session. There are 72 points in each panel. Each point represents the cumulative response totals up to and including the current trial. The slope of the function at any point in the scatterplot indicates response allocation. The diagonal line represents indifference, that is, equal responding to the two initial links. The data are from a representative subject (Pigeon 194).

To produce a more systematic analysis of changes in response allocation within individual sessions, we fit linear and bi-linear (i.e., 'broken stick') models to the cumulative scatterplots like those in Figure 5.4 for all subjects and sessions. The linear model was defined in terms of two parameters (i.e., $y = ax + b$); the bi-linear model had four parameters, as it consisted of two linear components. Estimates of parameters were obtained for all sessions and subjects using a nonlinear optimization procedure (Microsoft

Excel Solver). *F* ratios were then calculated to test whether the increased variance accounted for by the bi-linear over the linear model was significant ($p < .05$). If the incremental variance was significant, and visual inspection confirmed there were no additional changes in preference during the session, the session was determined to have a single change point. If the incremental variance was not significant, the session had no change point. According to these criteria, 79.4% of sessions had exactly one change point (see Figure 5.4, panels A and C), 16.7% had no change points (panel B), and the remaining 3.9% either had more than one visually identifiable change point (panel D) or changed too gradually for a single change point to be identified. Linear and bi-linear models accounted for a large percentage of variance, $M = .99$ [$SE = 0.00$] and $M = .99$ [$SE = 0.00$] for sessions classified as having zero and exactly one change point, respectively.

Next, for sessions with a single change point, we estimated the location of the change through linear interpolation. To illustrate, assume that the *x* coordinate (i.e., cumulative number of left initial-link responses) of the intersection point of the bi-linear function was 113.69, and that the pigeon had made 86 left initial-link responses by the end of the ninth trial and 118 by the end of the tenth trial. The 113.69^{th} response was then estimated to have occurred at 100*(113.69-86)/(118-86) = 86.5% of the way through the 10th trial; thus the change point was estimated as occurring after 9.87 trials.

Table 5.2 shows mean change points for bi-linear sessions for all subjects in both dominated and tradeoff sessions. For all subjects, change points for tradeoff sessions ($M = 30.76$, $SE = 1.85$) occurred on average later than those for dominated sessions ($M = 23.37$, $SE = 0.81$), $t(3) = -3.24$, $p < .05$. This suggests that the acquisition of preference occurred more rapidly in dominated than tradeoff sessions.

Table 5.2. Mean location (in number of trials) of the change point for each subject in the dominated and tradeoff sessions that were characterized by a single change point in Experiment 1.

	Dominated Sessions		Tradeoff Sessions	
Pigeon	Mean	SD	Mean	SD
191	24.08	10.08	33.29	17.34
192	24.90	13.25	25.46	15.43
193	21.11	12.30	30.91	17.69
194	23.38	09.28	33.37	20.27

Analyses of cumulative response scatterplots showed that, for all but 2 sessions of Experiment 1, response allocation within individual sessions could be characterized by zero, one, or multiple change points and that response allocation between change points was stable. This implies that, if all change points in the session occurred prior to the 61st trial, then responding in the last block of 12 trials (i.e., 61 through 72) should provide a good measure of response allocation in an individual session, that is, a measure suitable for comparing dominated and tradeoff sessions. Three sessions that had change points after the 60th trial were excluded from the subsequent analysis, as were the 2 sessions referred to earlier.

To verify that performance in the final block of sessions was stable, we compared response allocation in the first and second halves of the block (trials 61 to 66 and 67 to 72, respectively). For all subjects, dependent-means t-tests confirmed that response allocation did not change systematically over the last 12 trials.

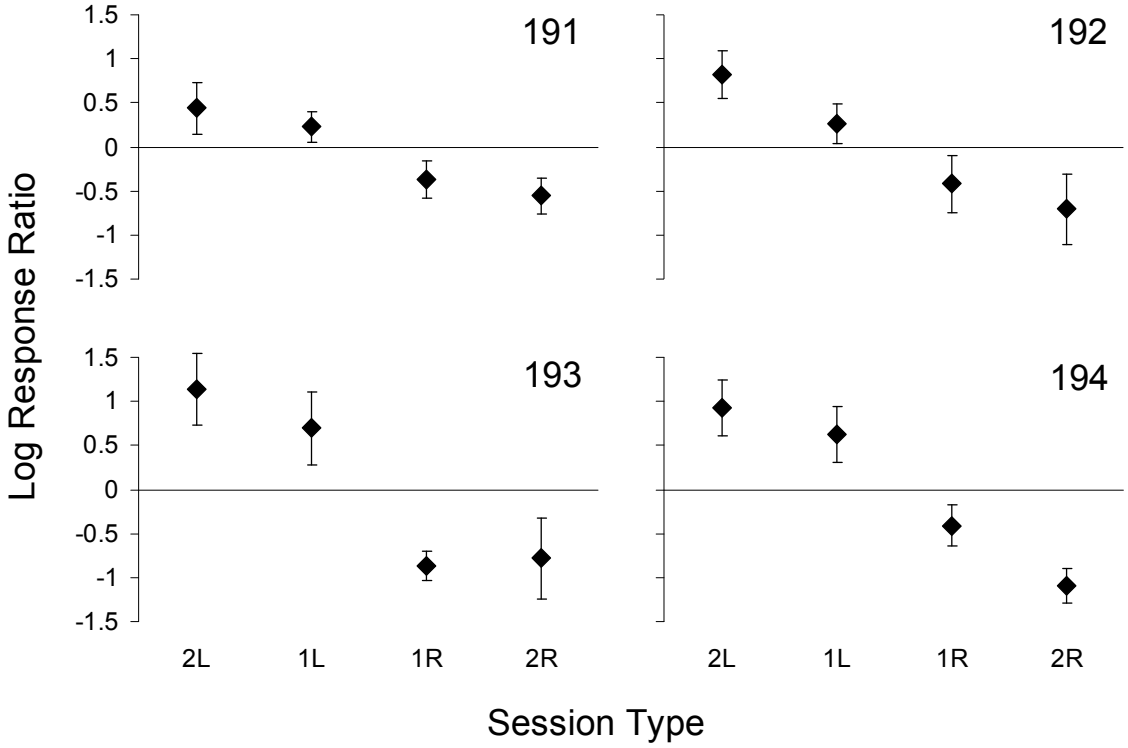

Figure 5.5. For each subject, mean log response ratios are shown from the final block according to session type. Error bars show standard deviation. See text for more details.

Figure 5.5 shows mean log response ratios from the final block of 12 trials from each session. The sessions were grouped as follows: Left-dominated sessions (i.e., the shorter delay and larger magnitude were both associated with the left alternative) in which response allocation favored the left alternative are labeled "2L". Tradeoff sessions are labeled "1L" and "1R" when response allocation favored the left and right alternatives, respectively. Right-dominated sessions in which response allocation favored the right alternative are labeled "2R". Pigeon 191 preferred the right alternative for a single left-dominated session; although this could be considered a "0R" session, it was excluded from the analysis. With one exception (for tradeoff sessions where Pigeon 193 preferred the right alternative), mean response allocation in tradeoff sessions was always less extreme than mean response allocation in dominated sessions. This finding is consistent with the

generalized matching law and suggests that response allocation in individual sessions is not determined by a single dimension selected stochastically each session.

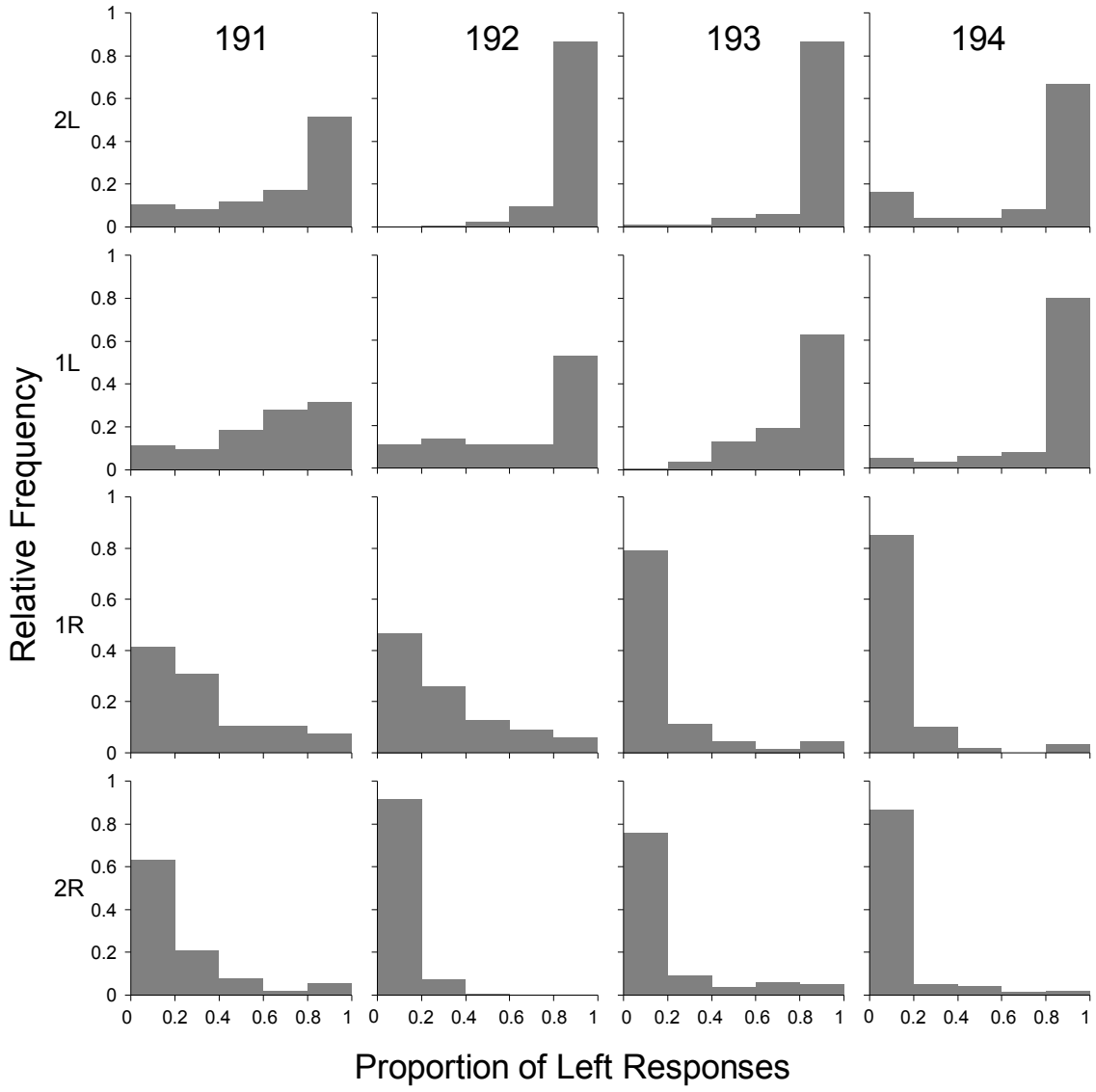

Figure 5.6. Relative frequency distributions of the proportion of left responses from each of the 61st through 72nd trials of all sessions included in Figure 5.5, for each subject and type of session.

However, the intermediate preferences obtained in tradeoff sessions could have resulted from the pigeons responding on the basis of a single reinforcer dimension on a more local timescale than the final block of trials. Specifically, if the controlling dimension changed during the final block, then an intermediate preference could be obtained in tradeoff compared to dominated sessions. Thus we examined performances on individual

trials during the final block of each session. If the intermediate preferences in tradeoff sessions resulted from shifts in the controlling dimension (even unsystematic ones), then the relative frequency distribution of choice proportions on single trials should show evidence of bimodality, with modes at either extreme.

Figure 5.6 shows relative frequency distributions of choice proportions (rather than log response allocation to avoid division by zero) for individual trials, binned in five intervals of width = .20, for each subject and session type. Distributions appear unimodal with greater variance in tradeoff than dominated sessions. To confirm quantitatively that the observed distributions in tradeoff sessions could not be described by extreme preference on individual trials, we conducted Kolmogorov-Smirnov tests comparing observed distributions to those predicted if the choices on individual trials were constrained to be extreme (i.e., in bins 0.0 to 0.2, and 0.8 to 1.0), but yielded the same overall obtained choice proportion. In all cases, the Kolmogorov-Smirnov tests showed that the obtained distributions were significantly different from those predicted (all p's < .001). This provides evidence that the intermediate preferences in tradeoff sessions were not produced by shifts in the controlling dimension across the last block of trials. Instead, results are consistent with the prediction of the concatenated matching law that response allocation is determined by multiple reinforcer variables.

5.4 Experiment 5: immediacy, magnitude and probability

In Experiment 5, reinforcer probability was changed across sessions, along with immediacy and magnitude, according to independent random sequences as in Experiment 1. We asked several questions: First, would there be evidence for control by all three multiple reinforcer dimensions at the molar level? If so, would performance in individual

sessions and cycles reveal control over responding by single or multiple reinforcer dimensions? Finally, we planned to test whether effects of immediacy, magnitude, and probability had additive and independent effects on response allocation, as required by the matching law, or whether there was evidence for an interaction.

5.4.1 Method

5.4.1.1 Subjects

The four pigeons from Experiment 4 served and were maintained under the same conditions.

5.4.1.2 Apparatus

The apparatus was that described for Experiment 4.

5.4.1.3 Procedure

The concurrent-chains procedure that was used was the same as in Experiment 4 with the exception that, in addition to immediacy and magnitude, the probability that a terminal link ended in reinforcement was varied across sessions. As in Experiment 4, the terminal-link FI schedule values were 8 s and 16 s, and the reinforcer magnitudes were 2 s and 4 s with the assignment of immediacy and magnitude values determined by independent random binary sequences. In addition, terminal links for one alternative were always reinforced (with a probability of 1.0) whereas terminal links for the other were reinforced with a probability of 0.5. The assignment of reinforcer probability was determined by a third independent random binary sequence. For the 0.5 probability terminal link, the first response after the FI schedule had elapsed produced either reinforcement or a blackout of equivalent duration, as determined by sampling a probability gate with $p = .50$.

There were eight possible configurations of sessions. Dominated sessions, where the shorter delay (FI 8 s), larger magnitude (4 s) and greater probability of reinforcement (1.0) were all assigned to the left or all to the right, comprised one quarter of all sessions on average. The six other possible session configurations were tradeoff sessions where shorter delay and larger magnitude, shorter delay and greater probability, or larger magnitude and greater probability were assigned to the same key (left or right). The expected reinforcer access for each terminal link in tradeoff sessions, calculated as expected seconds of access to food per terminal-link delay, was greater for the alternative associated with the richer option, that is, the option favored by two out of three dimensions than for the alternative favored by a single dimension only: 0.25 s and 0.125 s reinforcer access per 1 s of terminal-link delay, respectively.

Training in Experiment 5 lasted for 177 sessions and began immediately after the conclusion of Experiment 4. Based on visual inspection of plots of sensitivity to immediacy, magnitude, and probability across sessions (10- and 20- session equally-weighted moving averages) for each subject, the data included in analyses are from the last 100 sessions.

5.4.2 Results

Figure 5.7 shows the log initial-link response and terminal-link reinforcer access ratios (left key/right key) for all subjects across the last 100 sessions of Experiment 5. In Experiment 5, the log terminal-link reinforcer access ratio is the log ratio of the products of reinforcer magnitude, probability, and immediacy: $\log((M_L*P_L/D_L)/(M_R*P_R/D_R))$. Each data point represents performance in a single session. Overall, changes in response allocation appeared to be stable across the 100 sessions for all subjects. That is, similar to

Experiment 1, response allocation for all pigeons generally tracked changes in the reinforcer access ratio: Preference for the left alternative increased from one session to the next if the relative reinforcer access of the left terminal link increased and decreased if the relative reinforcer access for the left terminal link decreased.

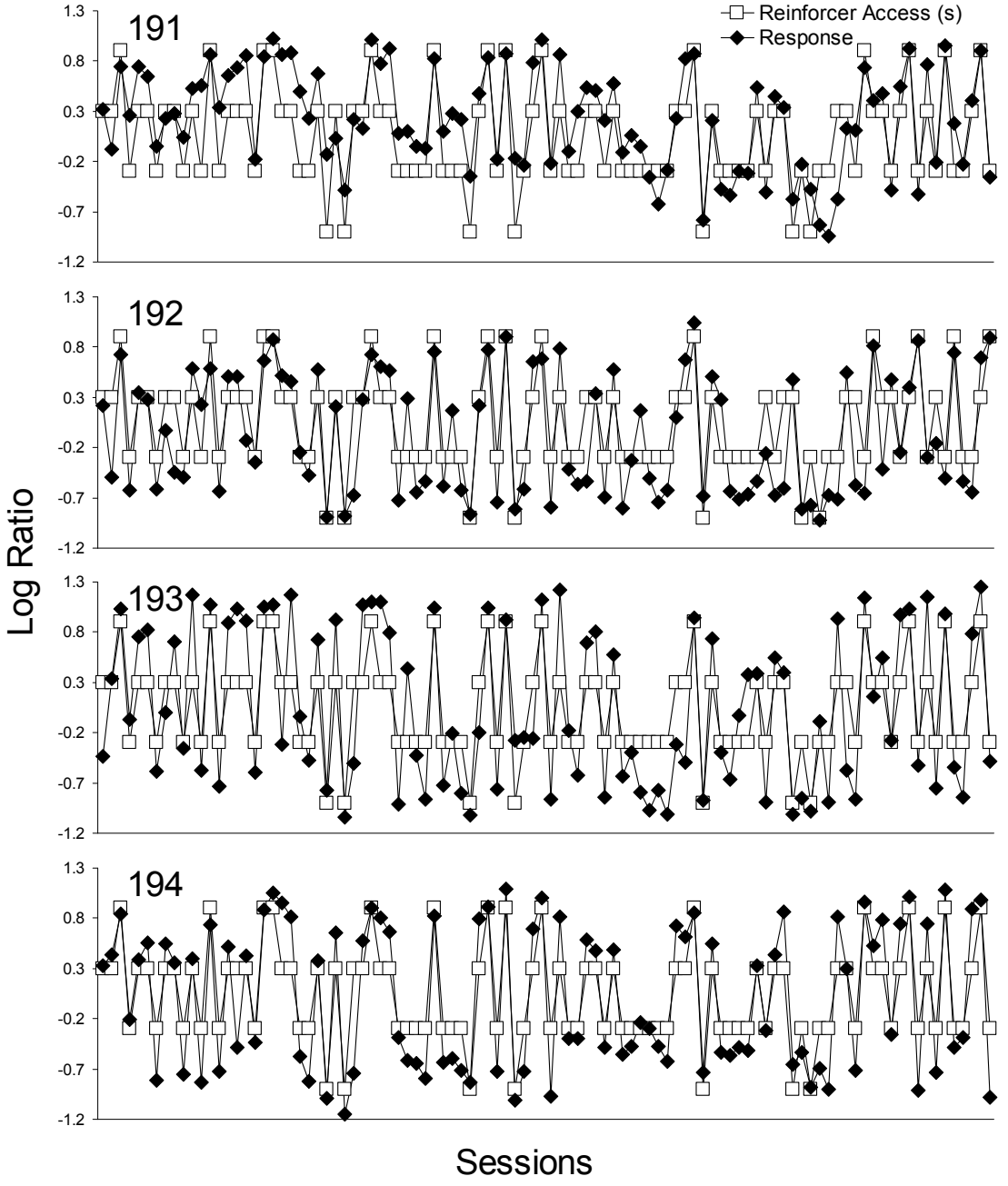

Figure 5.7. Log initial-link response ratios and log terminal-link reinforcer access ratios across the last 100 sessions of Experiment 5. Reinforcer access ratios were calculated as described in the text.

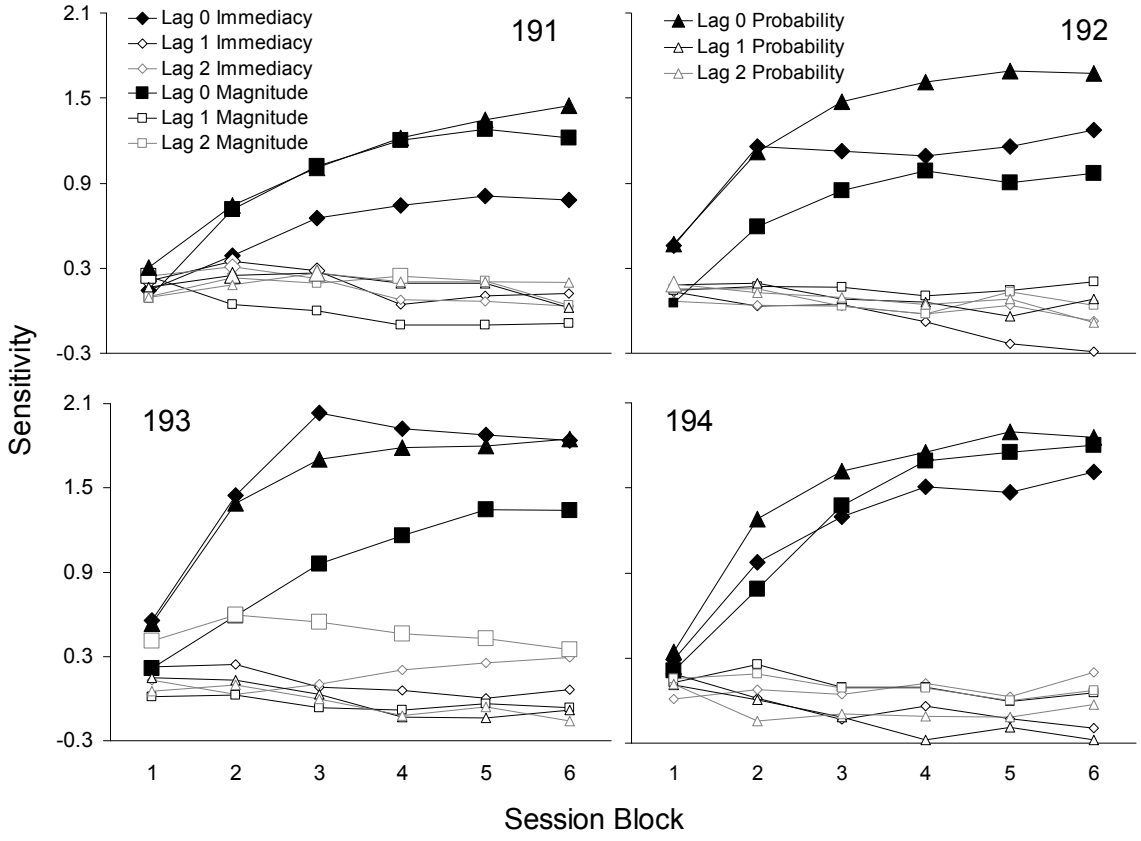

Figure 5.8. Sensitivity to log immediacy, magnitude and probability ratios for Lag 0 through Lag 2 for each block of 12 trials. Diamond data points represent immediacy ratios, square data points represent magnitude ratios, and triangles represent probability ratios. Larger data points represent statistically significant coefficients.

To assess quantitatively the degree of control of the current- and prior-session immediacy, magnitude, and probability ratios on response allocation, we applied a generalized-matching model to the data:

$$\begin{aligned}\log\frac{B_{0L}}{B_{0R}} = {} & a_{0D}\log\frac{1/D_{0L}}{1/D_{0R}} + a_{0M}\log\frac{M_{0L}}{M_{0R}} + a_{0P}\log\frac{P_{0L}}{P_{0R}} \\ & + a_{1D}\log\frac{1/D_{1L}}{1/D_{1R}} + a_{1M}\log\frac{M_{1L}}{M_{1R}} + a_{1P}\log\frac{P_{1L}}{P_{1R}} \\ & + a_{2D}\log\frac{1/D_{2L}}{1/D_{2R}} + a_{2M}\log\frac{M_{2L}}{M_{2R}} + a_{2P}\log\frac{P_{2L}}{P_{2R}} \\ & + \log b\end{aligned} \quad .$$

Equation 5.5

Equation 5.5 extends Equation 5.3 to include log probability ratios: *P* refers to probability, while other variables and subscripts are as in Equation 5.3. Figure 5.8 shows Lag 0-2 sensitivity coefficients for immediacy, magnitude, and probability for each block of 12 trials within a session. For all subjects, Lag 0 sensitivities for each reinforcer dimension increased over the course of the session, whereas those for higher lags decreased or remained near zero. There were no systematic differences in Lag 0 sensitivity coefficients by reinforcer dimension; for all subjects, strong control was evidenced by immediacy, magnitude, and probability. Lag 0 coefficients were statistically significant for all blocks for two subjects (191 and 194), and all but the first block for the other two subjects. There was no systematic control by immediacy, magnitude, or probability ratios from previous sessions. Averaged across subjects, Lag 0 sensitivity to immediacy in the second half of the session was 1.34 [SD = 0.47], Lag 0 sensitivity to magnitude was 1.30 [SD = 0.33], and Lag 0 sensitivity to probability was 1.66 [SD = 0.23].

To determine whether the effects of the reinforcer dimensions were additive and independent, we analyzed data from the second half of each session by applying a simplified generalized-matching model to the data:

$$\log\frac{B_{0L}}{B_{0R}} = a_{0D}\log\frac{1/D_{0L}}{1/D_{0R}} + a_{0M}\log\frac{M_{0L}}{M_{0R}} + a_{0P}\log\frac{P_{0L}}{P_{0R}} + \log b$$

Equation 5.6

To test whether any of the four interactions (that is, products of the values of the coefficients of sensitivity—(a) between immediacy and magnitude ratios, (b) between immediacy and probability ratios, (c) between magnitude and probability ratios, and (d) between immediacy, magnitude and probability ratios) were significant, we fit Equation 5.6 to the data from the second half of each session and then computed the incremental

variance accounted for by the addition of each interaction term individually. The results are shown in Table 5.3.

Table 5.3. Results of regressing log response allocation (averaged over the final three blocks of each session) on Lag 0 log immediacy, magnitude and probability ratios from Experiment 2. Sensitivity coefficients for immediacy, magnitude and probability are the unstandardized weights obtained prior to the addition of any interaction term. IxM, IxP, MxP, and IxMxP refer to the interactions and R^2_{inc} to the increment in R^2 when the interaction value was applied.
*** $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$.**

Pigeon	Immediacy	Magnitude	Probability	Intercept	R^2
191	0.75***	1.22***	1.31***	0.22***	0.75
192	1.21***	0.99***	1.66***	-0.13*	0.73
193	1.87***	1.32***	1.86***	-0.06	0.72
194	1.59***	1.76***	1.86***	-0.07	0.81

	Interaction							
Pigeon	IxM	R^2_{incIxM}	IxP	R^2_{incIxP}	MxP	R^2_{incMxP}	IxMxP	$R^2_{incIxMxP}$
191	0.39	0.00	-0.08	0.00	-0.19	0.00	-2.84*	0.01
192	0.09	0.00	0.49	0.00	0.55	0.00	-3.58*	0.02
193	-0.73	0.00	0.14	0.00	0.74	0.00	-9.82***	0.07
194	-0.25	0.00	-0.39	0.00	0.11	0.00	-11.49***	0.11

For all subjects, the three-way interaction was statistically significant. Specifically, the relative effect of any one reinforcer dimension was greater when the other two dimensions favored different alternatives than when both favored either the left or the right alternative. Figure 5.9 plots the average log response ratio from the second half of each session for each combination of immediacy and magnitude ratio when the log probability ratio was negative (left panels) and positive (right panels) for individual subjects. The relative effect of magnitude was greater when the signs of the log immediacy and probability ratios were mixed than when their signs were either both positive or both negative. The distance between filled and unfilled data points in Figure 5.9 illustrates the effect of magnitude on response allocation. For all subjects, that distance was greater for the two center pairs of data points than the left- and rightmost pairs (when immediacy and probability both favored the right and left alternatives, respectively). None of the two-way interactions were significant.

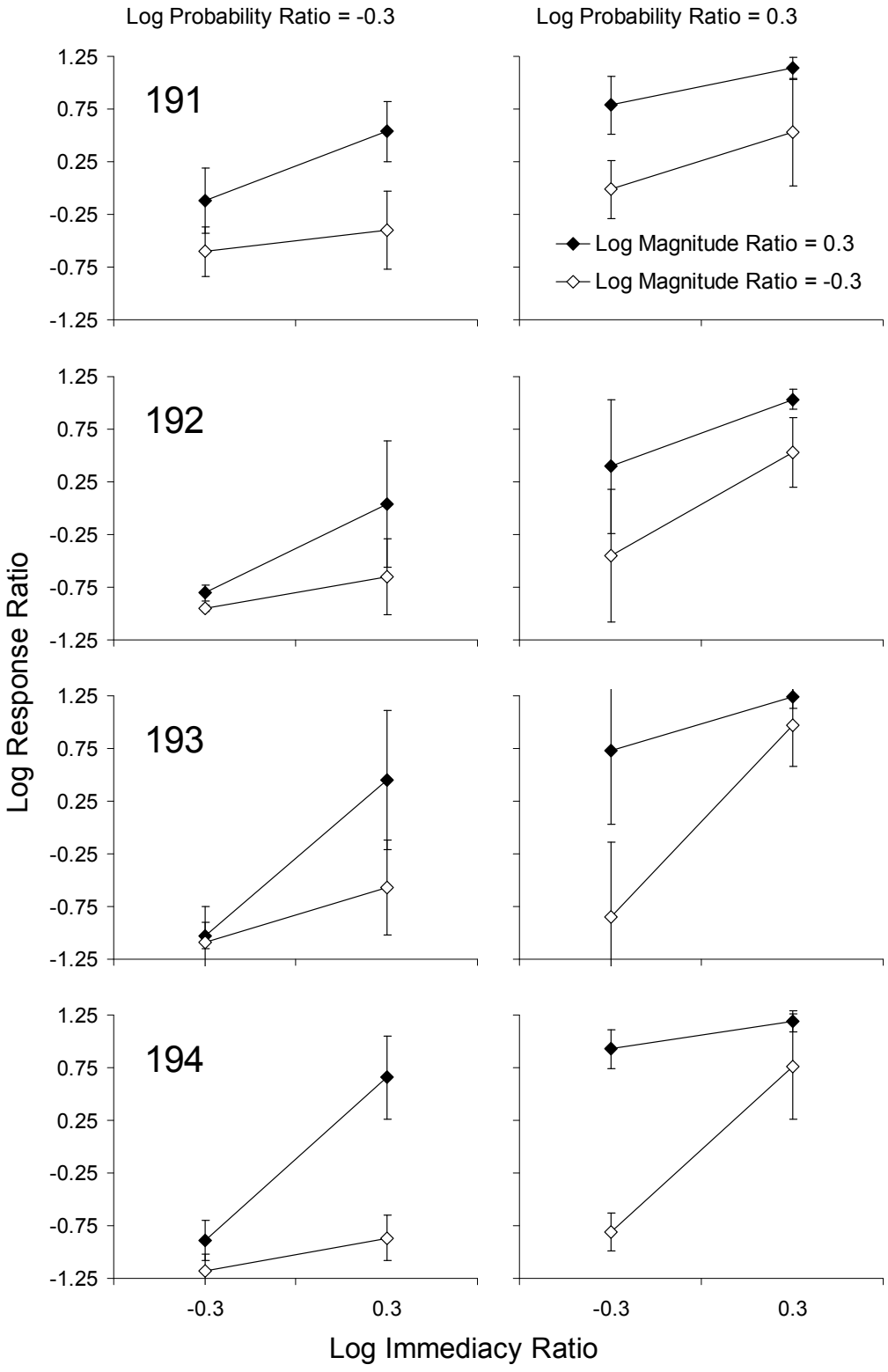

Figure 5.9. Mean log response ratio as a function of log immediacy ratio when log magnitude ratio was positive (filled data points) and negative (unfilled) and when log probability ratio was negative (left panels) and positive (right panels). Error bars represent the standard deviation.

Next we conducted analyses to test whether response allocation was less extreme in tradeoff compared to dominated sessions, as predicted by the concatenated generalized matching law. As in Experiment 4, we examined cumulative response scatterplots for individual sessions, calculated change points based on linear and bi-linear models, and compared response allocation in the first and second half of the last block of trials to confirm that log response allocation in the final block would be an appropriate measure of final preference for individual sessions in Experiment 5.

Table 5.4. Mean location (in number of trials) of the change point for each subject in the dominated and tradeoff sessions that were characterized by a single change point in Experiment 2.

	Dominated Sessions		Tradeoff Sessions	
Pigeon	Mean	SD	Mean	SD
191	23.19	7.80	29.25	12.77
192	15.80	7.21	23.49	14.95
193	21.27	14.98	26.06	14.50
194	20.92	10.96	26.21	11.63

Cumulative response scatterplots were similar to those observed in Experiment 4, that is, changes in preference tended to be abrupt. 86.5% of sessions had exactly one change point, 7.5% had none, 5.25% had more than one visually identifiable change point, and 0.75% (three sessions) changed too gradually for a single change point to be identified. Linear and bi-linear models accounted for a high average proportion of variance, $M = .99$ [$SE = 0.00$] and $M = .99$ [$SE = 0.00$] for sessions classified as having zero and exactly one change point, respectively. Table 5.4 shows mean change points for bi-linear sessions each pigeon. Change points in tradeoff sessions ($M = 24.80$, $SE = 1.07$) occurred later on average than change points in dominated sessions ($M = 21.38$, $SE = 1.80$), but the result was not statistically significant, $t(3) = 1.70$, $p > .05$.

We excluded nine sessions that had change points after the 60th trial from all subsequent analyses. To verify that performance in the final block of the remaining

sessions was stable, we compared response allocation in the first and second half of that block. For all subjects, dependent-means t-tests confirmed that response allocation did not change over the last 12 trials. Thus, response allocation was not changing systematically during the final block.

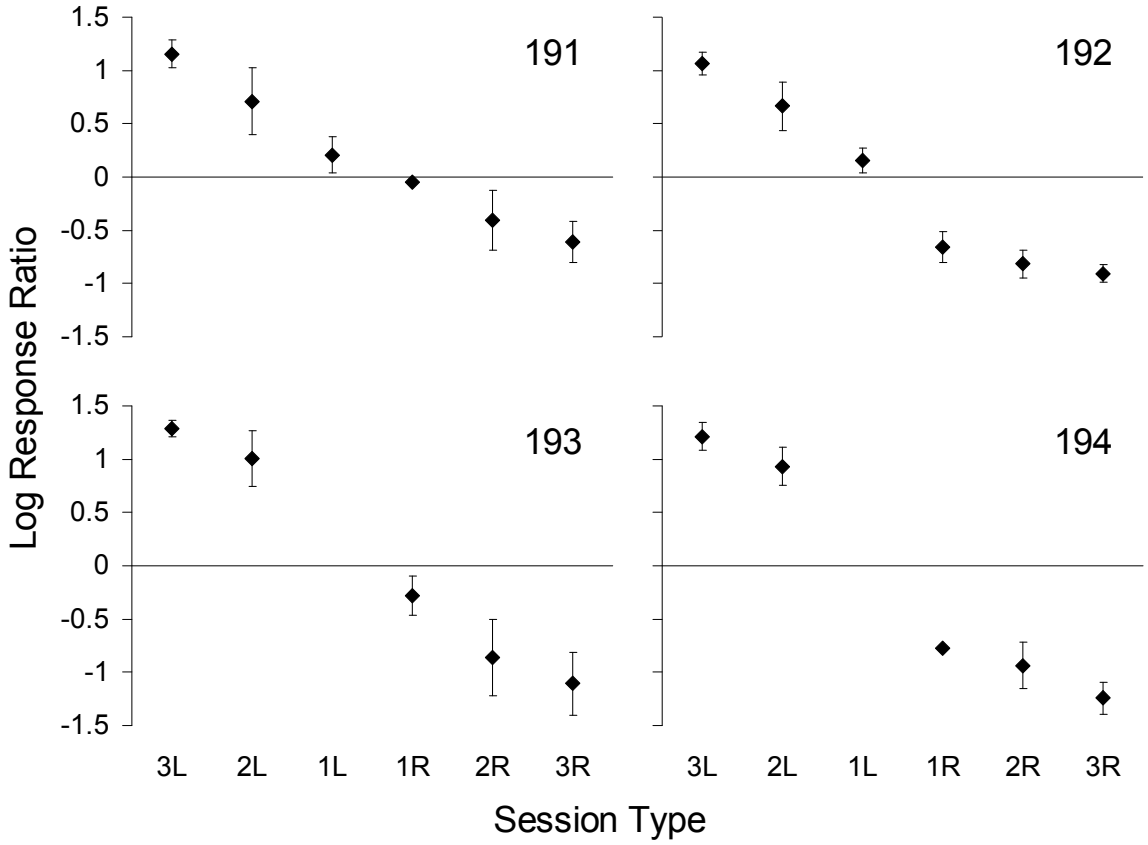

Figure 5.10. For each subject, mean log response ratios are shown from the final block of trials for each session type. Error bars show standard deviation. See text for more details.

Figure 5.10 shows mean log response ratios from the final block of trials for sessions grouped according to whether more responses were made to the left or right alternative and the number of reinforcer dimensions that favored that alternative. Left-dominated sessions in which response allocation favored the left alternative are labeled "3L." Tradeoff sessions are labeled "2L" or "1L" and "2R" or "1R" when response allocation favored the left and right alternatives (and, respectively, two or one reinforcer dimensions favored the pigeon's preferred alternative), respectively. Right-dominated

sessions in which response allocation favored the right alternative are labeled "3R." Mean response allocation in tradeoff sessions was less extreme than mean response allocation in dominated sessions in all 15 cases, consistent with the possibility that multiple reinforcer dimensions control choice within individual sessions.

Figure 5.11 shows relative frequency distributions of choice proportions (rather than log response allocation to avoid division by zero) for individual trials, binned in five intervals of width = .20, for each subject and session type. Distributions appear unimodal with the intermediate levels of preference in tradeoff compared to dominated sessions attributable to greater variability. To confirm quantitatively that the observed distributions of choice proportions in tradeoff sessions could not be described by extreme preference on individual cycles, we conducted Kolmogorov-Smirnov tests comparing observed distributions to those predicted if the choices on individual trials were constrained to be extreme (i.e., in bins 0.0 to 0.2, and 0.8 to 1.0), but yielding the same overall obtained choice proportion. In all cases, the Kolmogorov-Smirnov test showed that the obtained distributions were significantly different from the predicted distributions (all p's < .01). This provides evidence that the intermediate preferences in tradeoff sessions were not produced by shifts in the controlling dimension across the last block of trials. Thus, the intermediate preferences suggest control by multiple reinforcer dimensions within single sessions, as predicted by the concatenated matching law and similar to results from Experiment 4.

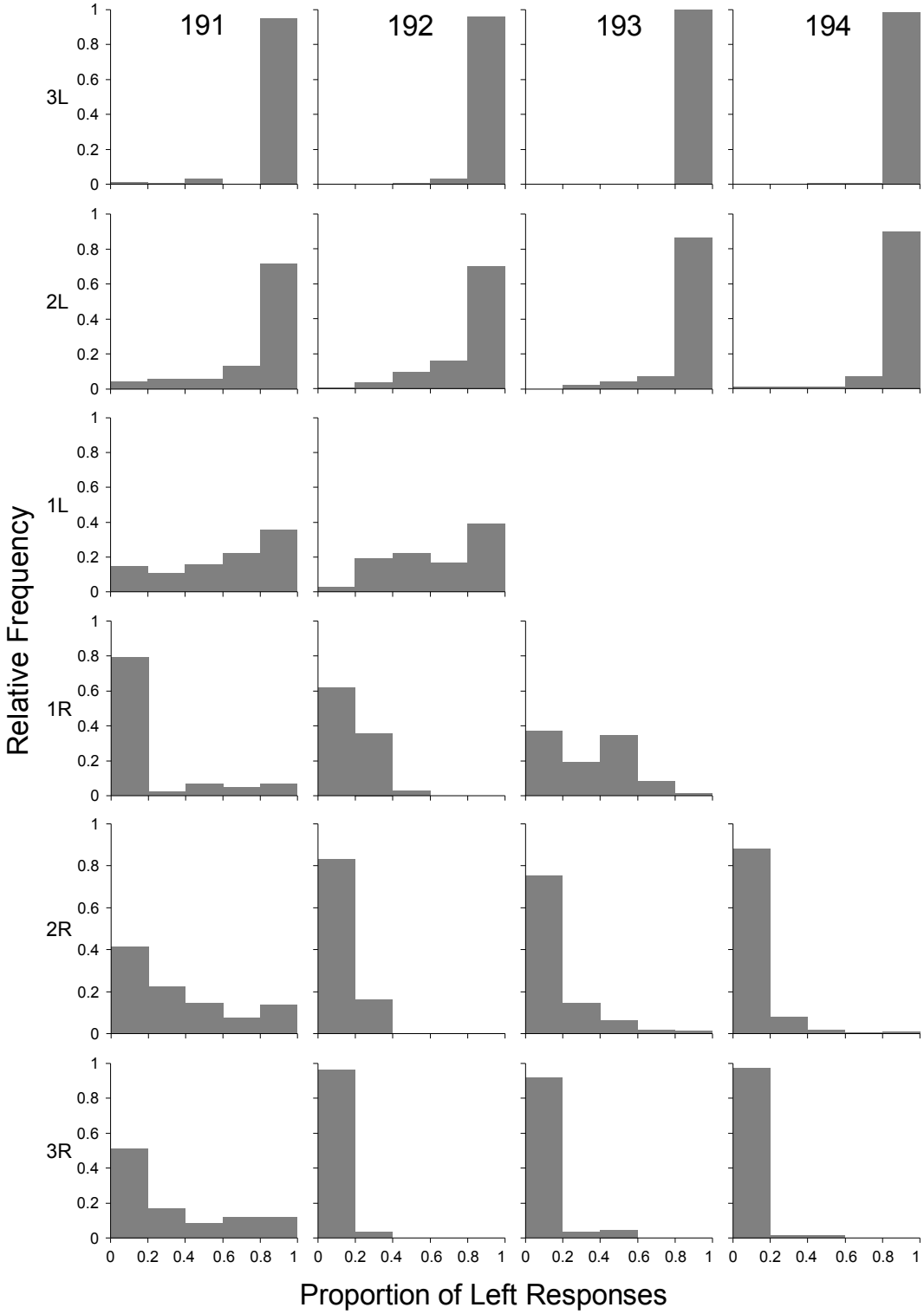

Figure 5.11. Relative frequency distributions of the proportion of left responses from each of the 61st through 72nd trials of all sessions included in Figure 5.10, for each subject and type of session.

5.5 General discussion of Experiments 4 and 5

The first question addressed by the present research was whether pigeons' response allocation in a concurrent-chains procedure in which terminal links differed on multiple reinforcer dimensions which changed unpredictably across sessions (cf. Grace et al., 2003) would show sensitivity to multiple dimensions, similar to findings from steady-state research (e.g., Grace, 1995). Both experiments provided substantial evidence of control by multiple reinforcer dimensions at the molar level. In Experiment 4, multiple regression analyses showed that sensitivities to both log immediacy and magnitude ratios from the current session (i.e., Lag 0) were positive and statistically significant, whereas sensitivities for prior sessions were near-zero and generally nonsignificant, especially by the final blocks of sessions. In Experiment 5, terminal link immediacy, magnitude, and probability of reinforcement were varied across sessions. To our knowledge, this is the first occasion when three reinforcer dimensions have been manipulated simultaneously in research on behavioral choice. Multiple regression analyses of Experiment 5 showed that Lag 0 sensitivities for immediacy, magnitude and probability were positive and statistically significant, whereas those for higher lags were near-zero and nonsignificant. These results extend those of previous studies by demonstrating that pigeons' response allocation in concurrent chains can be controlled by differences in multiple reinforcer dimensions when alternatives change unpredictably across sessions.

Two possible underlying processes that might produce sensitivity to multiple reinforcer dimensions at the molar level were described in the introduction. One is that reinforcer ratios combine additively and independently to determine reinforcer value and response allocation in any temporal epoch, according to the concatenated matching law.

Alternatively, a single reinforcer dimension might determine response allocation in a given epoch but the controlling dimension could change across epochs.

To distinguish between these possibilities using the individual session as epoch, it was necessary to examine the level of preference in individual sessions during the final block of trials. To accomplish this, sessions were classified as either *dominated* – those in which all reinforcer dimensions favored the same alternative – or *tradeoff* – those in which at least one dimension favored each alternative. Because response allocation virtually always favored the richer alternative in dominated sessions, the critical comparison involved the degree of preference for the left (or right) alternative in left- or right-dominated sessions and that reached in tradeoff sessions when the left or right alternative was favored. If a single reinforcer ratio determined preference in any individual session, preference in dominated sessions should not be more extreme than preference in tradeoff sessions in which the subject favored the same alternative. By contrast, the concatenated matching law predicts that preference should be less extreme in tradeoff sessions. In both experiments, response allocation in tradeoff sessions was generally less extreme than in dominated sessions. Moreover, in Experiment 5, the strength of response allocation for an alternative during tradeoff sessions was positively correlated with the number of dimensions that favored that alternative. These results suggest that multiple dimensions controlled response allocation within individual sessions.

To distinguish between the possibilities using the individual trial as epoch, we inspected distributions of choice proportions calculated over individual trials from the final block of trials for bimodality. If a single reinforcer ratio determined preference in individual trials, choice proportions should always be extreme, and the intermediate levels

of preference observed in tradeoff relative to dominated sessions would be produced by different combinations of extreme preference for the left alternative and extreme preference for the right. That is, the distribution of choice proportions should be bimodal. By contrast, if relative value determined preference in individual trials according to the concatenated matching law, the distribution of choice proportions should be unimodal. Distributions were unimodal in both experiments. These results suggest that multiple dimensions controlled choice proportion within individual cycles.

The question of whether multiple or single reinforcer dimensions control responding in a given temporal epoch has parallels in research on human judgment and decision making. One major class of models for judgment and decision making is referred to as "information integration," because models assume that values on different attributes are combined before a decision is made (Anderson, 1968; Massaro & Friedman, 1990). For example, Anderson (1962) showed that different personality traits combined additively and independently when human participants formed impressions about others based on verbal descriptions. By contrast, heuristic models that assume choice is based on "fast and frugal" strategies (e.g., Gigerenzer & Goldstein, 1996) provide an alternative account of decision making. According to single-cue heuristic strategies, all-or-none decisions can be made on the basis of a single attribute, even though other information may be available. The concatenated matching law is an information integration model because relative value is determined by multiple reinforcer dimensions. The hypothesis that one dimension at a time controls responding shares critical features with single-cue heuristics, namely, that decisions are all-or-none and based on a single attribute, regardless of which alternative has greater utility when all dimensions are considered.

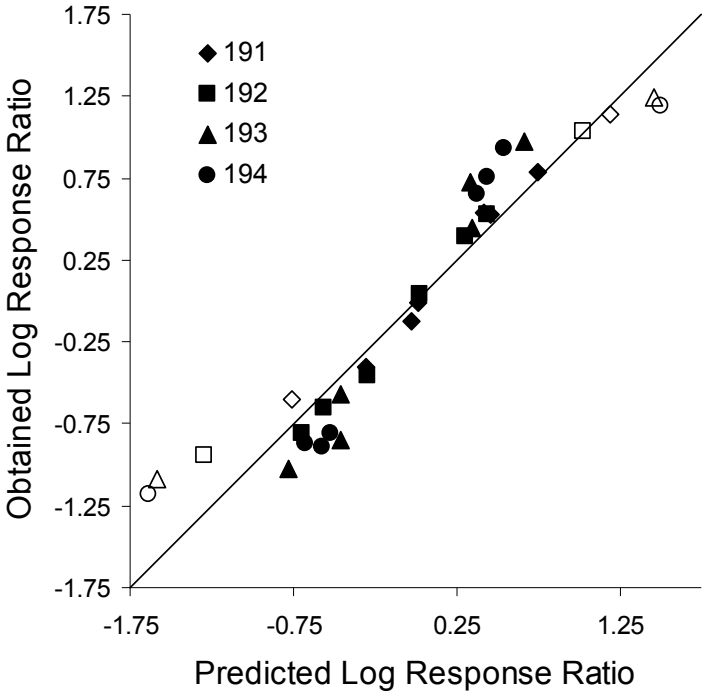

Figure 5.12. Mean log response ratios for the second half of trials in the eight types of sessions from Experiment 5. Values predicted by the concatenated generalized matching law (Equation 5.6) are plotted on the x-axis and obtained values on the y-axis. The unfilled data points represent dominated sessions, the filled tradeoff sessions.

The difference between response allocation during dominated and tradeoff sessions was consistent with the view that subjects were integrating information from multiple reinforcer dimensions within individual sessions. But can this integration process be adequately described by the concatenated generalized matching model? According to the generalized matching model (Equation 5.6), effects of different reinforcer dimensions on response allocation should be additive and independent. Contrary to this assumption, there was a significant three-way interaction (immediacy x magnitude x probability) for all subjects in Experiment 5. How is this interaction to be interpreted? Figure 5.12 shows, for all subjects, the mean obtained log response ratio from the last three blocks of trials within sessions in Experiment 5 for each of the eight session configurations (i.e., immediacy, magnitude, probability favored the left alternative; immediacy and magnitude favored the left alternative while probability favored the right; etc.) as a function of the log response

ratio predicted by Equation 5.6. Figure 5.12 shows clearly that the relationship is sigmoidal: Response allocation was less extreme than predicted in dominated sessions (7 out of 8 cases) and more extreme than predicted in tradeoff sessions (23 out of 24). Thus, the three-way interaction in Experiment 5 is consistent with a sigmoidal relationship between relative value (as calculated by Equation 5.6) and response allocation.

One possible cause of this nonlinearity is a ceiling effect caused by some aspect of the procedure such as the dependent scheduling of terminal links. Dependent scheduling makes each trial within a session essentially a forced choice and could produce a ceiling effect because it places constraints on how extreme response allocation can be if the subject is to complete all cycles in a fixed period of time. Previous research with rapid acquisition concurrent chains has also used dependent scheduling and similarly found evidence of a sigmoidal relationship between response allocation and relative immediacy (Grace et al., 2003; Grace & McLean, 2006; Kyonka & Grace, 2007).

Table 5.5. Results of fitting a logistic function (Equation 5.7) to the predicted values in Figure 5.12. Parameters c, *d*, and τ were estimated for each subject using a nonlinear optimization procedure. R^2 represents the variance accounted for by Equation 5.7, R^2_{inc} represents the improvement in variance accounted for by Equation 5.7 over a linear model (Equation 5.6). Range refers to the degree of shift in preference, that is, to the difference between the mean log response ratio in left-dominated sessions and the mean log response ratio in right-dominated sessions.

	Logistic Parameters					
Pigeon	*d*	*c*	τ	R^2	R^2_{inc}	Range
191	-1.54	3.12	0.71	0.98	0.01	1.74
192	-1.07	2.15	0.37	1.00	0.04	1.98
193	-1.17	2.40	0.32	0.99	0.10	2.34
194	-1.17	2.39	0.31	1.00	0.11	2.37

If the sigmoidal relationship is due to a ceiling effect, then data for those pigeons that showed relatively greater range in obtained preference (i.e., the difference between the maximum and minimum log response ratios in Figure 5.12) should show the greater nonlinearity as well. The reason is this: Results for pigeons with preferences that were relatively more extreme in both directions (thus the greater range) should be more affected

by the flattening imposed by the ceiling effect. Results for pigeons with less extreme preferences would be less affected. If so, there should be a positive correlation between degree of nonlinearity and the range of preference. To assess the degree of nonlinearity in Figure 5.12, we fit a three-parameter logistic function to the data for individual subjects:

$$\log \frac{B_L}{B_R} = \frac{c}{1 + e^{-x/\tau}} + d \ ,$$

Equation 5.7

where $\log B_L/B_R$ is predicted log response ratio, x is the log response ratio predicted by Equation 5.6 (the x-axis in Figure 5.12), and c, d and τ are parameters. The degree of nonlinearity was assessed as the improvement in variance explained by Equation 5.7 over that associated with a two-parameter linear regression. Table 5.5 shows the resulting parameter estimates, percentages of variance accounted for, and the range of preferences. The improvement in fit by the logistic was negligible for Pigeon 191, small-to-moderate for Pigeons 192 and 193, and reasonably large for Pigeon 194. The range of preference shift was also greatest for Pigeons 193 and 194. Overall, there was a significant positive correlation between the improvement in variance accounted for by the logistic and range of preference shift, $r = 0.99$, $p < 0.01$. This result is consistent with the hypothesis that the sigmoidal relationship in Figure 5.12 is due to a ceiling effect. If that is the case, the observed nonlinearity is a potential methodological issue rather than a theoretical problem for the concatenated matching law. However, it is important to note that even if a ceiling effect was present and contributed to the nonlinearity, it would not necessarily imply that the ceiling effect was solely responsible for the three-way interaction obtained in Experiment 5. Other sources of nonlinearity might have been present.

If the sigmoidal relationship that appears in Figure 5.12 is not due to a methodological artifact, it disconfirms the concatenated matching law (Equation 5.6) as an adequate account of choice between alternatives that differ on multiple reinforcer dimensions. The basic assumptions of the matching law are that effects of different reinforcer dimensions on response allocation are additive and independent, and that the relationship between response allocation and relative reinforcer value is linear (in logarithmic terms) for each dimension. The linearity implies that the relationship between response allocation and relative value is described as a power function for each reinforcer dimension. The results in Figure 5.12 suggest that these assumptions may not be valid over the full range of reinforcer value.

Grace and McLean (2006) recently proposed a decision model to account for within-session changes of response allocation in concurrent chains. Their model may suggest an alternative explanation for the nonlinearity in Figure 5.12. According to their model, the strength of responding to one or the other of the initial links increases or decreases after a reinforcer has been received during a terminal link. Whether response strength increases or decreases depends on a comparison of the previous delay to reinforcement in that terminal link with a criterion that represents the delays experienced on both alternatives. The accuracy of these comparisons is determined by a parameter that represented the standard deviation of a log-normal distribution. Grace and McLean showed that, when the standard deviation was relatively large, predicted response allocation was a linear function of the log immediacy ratio, whereas when the standard deviation was relatively small, predicted response allocation was a sigmoidal function of the log immediacy ratio. However, their model was only developed to account for preference

between terminal links that differ in reinforcer immediacy and so would be need modification to incorporate the effects of reinforcer magnitude and probability.

It is possible to consider laboratory experiments like those in the present study simplified and controlled analogues of situations humans and other animals experience in everyday life. In most choice experiments, a single reinforcer dimension is under scrutiny at any given time, and conditions do not change until the subject's behavior is demonstrably stable. Because humans, pigeons, and other species often face decisions where the available options change from day to day and differ in terms of multiple dimensions, experiments like those reported here are arguably more ecologically valid than research using traditional steady-state procedures.

The present experiments show that pigeons' response allocation can adjust rapidly to frequent simultaneous changes in reinforcer immediacy, magnitude, and probability. It is important to emphasize that regardless of the specific nature of the process whereby response allocation changed within individual sessions, the results of that process, at the molar level, were highly adaptive. The pigeons in Experiment 5 faced an uncertain environment in which sources of reinforcement differed on three dimensions. Their aggregate responding showed that each dimension was weighted about equally in determining preference. Arguably, the pigeons behaved like "intuitive statisticians," calculating overall value in terms of the contributions from each dimension (cf. Anderson, 1968; Massaro & Friedman, 1990). By manipulating the validity of the dimensions, involving more than two possible ratios for each dimension, or examining choice between more than two alternatives, future research involving the rapid acquisition procedure should further illuminate the process that determines response allocation in concurrent-chains

procedures. Whatever the underlying process, our results suggest that multiple dimensions controlled preference in individual temporal epochs.

5.6 Notes on Experiment 6

Experiment 6, the final experiment presented in this thesis, was published as "The matching law and effects of reinforcer rate and magnitude on choice in transition" in the Society for Quantitative Analyses of Behavior special issue of *Behavioural Processes, 78*, 2008. Citations of Kyonka (2008) in Chapter 3 refer to this article. Four pigeons responded in a minimal-variation concurrent schedule in which reinforcer rate and magnitude were changed across sessions according to independent, random binary sequences. Goals of this experiment were to characterize response allocation and determine whether performance in Experiments 4 and 5 generalized to concurrent-schedule procedures. Overmatching is the typical result of concurrent-chains experiments with FI terminal links (Omino & Ito, 1993). If results were similar in a concurrent-schedule procedures previously associated with undermatching (Maguire, Hughes & Pitts, 2007; Schofield & Davison, 1997), would they be as detectable?

5.7 Experiment 6: The matching law and effects of reinforcer rate and magnitude on choice in transition

5.7.1 Abstract

Four pigeons responded in a concurrent-schedule procedure in which reinforcer rates and magnitudes changed unpredictably across sessions according to independent random series. Programmed relative reinforcement rates and magnitudes were always either 2:1 or 1:2.

Pigeons' response allocation tended to stabilize within sessions and multiple regression analyses showed that it was determined by rates and magnitudes from the current session. Sensitivity coefficients were positive and statistically significant for current-session reinforcement and magnitude ratios. Although there were individual differences in sensitivity to rate and magnitude, their interaction was not significant across subjects. Rate and magnitude both controlled responding in single sessions and individual interreinforcer intervals. Analyses of responding within sessions showed that preference was more extreme when the richer rate and larger magnitude were associated with the same alternative than when they were associated with different alternatives. Overall, results support the concatenated generalized matching law's assumptions of additivity and independence as applied to choice in transition.

Keywords: acquisition, choice, concatenated generalized matching law, reinforcement rate, reinforcer amount or magnitude, sensitivity, pigeons

5.7.2 Introduction

Concurrent variable-interval (VI) VI schedule experiments showing that preference is sensitive to reinforcement rate (Herrnstein, 1961) and reinforcer magnitude (Catania, 1963) were among the earliest studies of behavioral choice. Baum and Rachlin (1969, p. 870) proposed that when rate, magnitude and other independent variables are manipulated together, a multiplicative combination of their ratios might determine

preference. The concatenated generalized matching law (Davison and McCarthy, 1988) provides a quantitative framework for describing choice when outcomes differ on multiple reinforcer dimensions:

$$\log\left(\frac{B_L}{B_R}\right) = \left[\sum_{i=1}^{n} a_i \log\left(\frac{X_{iL}}{X_{iR}}\right)\right] + \log b\,.$$

Equation 5.8

In Equation 5.8, B_L and B_R are responses to left and right alternatives, respectively, X_{iL} and X_{iR} are the values of *i*th reinforcer dimension X_i associated with left and right alternatives, respectively, a_i is sensitivity to the *i*th dimension and log *b* represents bias. Equation 5.8 states that an additive, independent concatenation of the log ratios of different reinforcer dimensions determines log response allocation. It provides a good description of final, stable preference achieved in steady-state choice procedures in which reinforcer magnitude and rate (McLean and Blampied, 2001; Todorov 1973; but cf. Davison, 1988) or immediacy (Grace, 1995; Rachlin and Green, 1972) were varied across conditions.

Concurrent VI-VI schedule experiments showing that response allocation is sensitive to unpredictable changes in reinforcement rate (Schofield and Davison, 1997) and reinforcer magnitude (Maguire et al., 2007) were among the first studies to investigate choice in transition using procedures in which reinforcer contingencies changed across sessions. In similar concurrent chains experiments, pigeons' preference adapted to unpredictable changes in terminal-link delay to reinforcement and stabilized about halfway through sessions in which 72 reinforcers were delivered (Grace et al., 2003; Grace and McLean, 2006; Kyonka and Grace, 2007). Kyonka and Grace (2008) proposed that when multiple reinforcer dimensions are manipulated, the concatenated generalized matching law applies to choice in transition. They trained pigeons in two concurrent chains experiments.

In Experiment 1, relative immediacy and magnitude ratios were 2:1 or 1:2. In Experiment 2, relative immediacy, magnitude and probability ratios were 2:1 or 1:2. Each session, ratios were determined according to independent random series. Sessions were called "dominated" if the valances of all log ratios were the same: either positive (all reinforcer dimensions favored the left) or negative (all favored the right). "Tradeoff" sessions were those in which the valences were mixed: at least one dimension favored each alternative.

Kyonka and Grace (2008) regressed log initial-link response ratios on log immediacy, magnitude, and probability ratios from the current session (Lag 0) and two prior sessions (Lags 1 and 2). Whereas sensitivity coefficients for higher lags were close to zero, all Lag 0 sensitivity coefficients were positive and statistically significant, and they increased over the course of the session, indicating response allocation was determined by reinforcer ratios in the current session in that it was a function of the current session's reinforcement contingencies but not a function of contingencies in previous sessions. Overall, multiple-regression results supported the concatenated generalized matching law's assumption that the effects of different reinforcer dimensions are additive and independent.

Kyonka and Grace (2008) evaluated the applicability of the concatenated generalized matching law to performance within individual sessions and initial links by contrasting a concatenated-matching hypothesis (multiple reinforcer dimensions simultaneously combine to determine value) with the possibility that a single, stochastically-selected dimension controls response allocation, but the controlling dimension changes over some temporal epoch. They reasoned that if pigeons responded on the basis of a single dimension, the valence of any nonselected dimension(s) should have had no effect, so preference in individual temporal epochs – perhaps sessions or initial links

– always should have been extreme because the relative value of the preferred alternative was always the same. This was not the case. Response allocation in tradeoff sessions was less extreme than in dominated sessions. The intermediate levels of preference obtained in tradeoff sessions were not produced by a combination of extreme preferences for different alternatives. If they had been, distributions of choice proportions from individual initial links in tradeoff sessions would show evidence of bimodality; however, all distributions were unimodal with greater variability in tradeoff than dominated sessions. Consistent with the predictions of the concatenated matching law, multiple reinforcer dimensions determined response allocation that adapted to unpredictable changes.

The present study tests the generality of Kyonka and Grace's (2008) results with a different choice procedure and reinforcer dimension. Pigeons responded in concurrent VI-VI schedules in which programmed relative reinforcer rates and magnitudes were 2:1 and 1:2 and each session reinforcement and magnitude ratios were determined according to independent random series. The goals were to determine whether the effects of reinforcer rate and magnitude are additive and independent, and whether they can control responding within individual sessions and interreinforcer intervals.

5.7.3 Method

5.7.3.1 Subjects

Four pigeons of mixed breed and sex were maintained at 85% *ad libitum* weight plus or minus 15g through appropriate post-session feedings. Pigeons were housed individually in a vivarium with a 12-hr:12-hr light:dark cycle and free access to water and grit. All had experience with choice procedures but none had served previously in experiments in which contingencies changed unpredictably across sessions.

5.7.3.2 Apparatus

Four operant chambers (32 cm deep x 34 cm wide x 34 cm high) were enclosed in sound-attenuating boxes containing ventilation fans. Each chamber contained three keys 21 cm above the floor arranged in a row 10 cm apart, a houselight located above the center key and a grain magazine with a 5 x 5.5 cm aperture that was centered 6 cm above the floor. The houselight provided general illumination at all times except during reinforcer delivery. The magazine, which was illuminated during reinforcement, contained wheat. A force of approximately 0.15 N was necessary to operate each key. Experimental events were controlled through a microcomputer and MED-PC® interface located in an adjacent room.

5.7.3.3 Procedure

Because all pigeons had previous experience, concurrent-schedules training began immediately. Sessions ended after 72 reinforcers were delivered or 70 minutes had elapsed, whichever came first. Side keys were lighted white except during reinforcer delivery. When an interval selected from a VI 20-s schedule constructed from an exponential progression (Fleshler and Hoffman, 1962) had elapsed, the computer made a reinforcer available on the left or right key according to a probability gate programmed to assign two-thirds of reinforcers to the same key. The next peck to the assigned side key produced reinforcement if it satisfied a 1-s changeover delay. Reinforcer magnitudes were always 2-s and 4-s access to wheat. Therefore, both reinforcement ratios and reinforcer magnitude ratios were either 2:1 or 1:2. Across sessions, ratios changed according to independent random binary sequences, which were determined by downloading two series of random digits generated by a radioactive decay process (retrieved on July 14, 2003 from www.fourmilab.ch/hotbits/). The four possible configurations of rate and magnitude valences occurred with equal probability: A random half of the sessions were “dominated”

– both the richer rate and larger magnitude were associated with the left or both with the right alternative. The remaining half were "tradeoff" sessions in which the shorter delay was associated with one alternative and the larger magnitude with the other.

The experiment lasted 126 sessions. For all subjects, sensitivity to current-session log reinforcement and magnitude ratios, measured across blocks of 21 sessions, stopped changing systematically after the 3rd block of sessions. Therefore, the data included in these analyses are from the last 63 sessions.

5.7.4 Results

Table 5.6. Sensitivity coefficients (unstandardized weights, with standard error in parentheses) obtained from regressing log response allocation from the final three blocks of each session on Lag 0-2 log reinforcement and magnitude ratios. Coefficients represent parameter estimates for the *a* terms in Equation 5.9; for Pigeons 195, 196, 197 and 198 respectively, bias (log*b*, with standard error in parentheses) was 0.00 (0.18), 0.02 (0.01), -0.08 (0.01)* and 0.29 (0.03)***.**
*** *p* < .05. *** *p* < .001.**

Pigeon	Lag 0	Lag 1	Lag 2
Rate			
195	0.75(0.07)***	-0.04(0.07)	-0.01(0.03)
196	0.33(0.05)***	0.07(0.05)	0.01(0.04)
197	0.73(0.06)***	0.15(0.06)*	0.06(0.06)
198	0.47(0.10)***	0.09(0.10)	-0.01(0.10)
Magnitude			
195	0.66(0.06)***	0.09(0.07)	-0.07(0.07)
196	0.22(0.05)***	-0.03(0.05)	-0.05(0.05)
197	0.48(0.07)***	0.04(0.06)	-0.03(0.06)
198	1.00(0.10)***	0.02(0.10)	0.14(0.10)

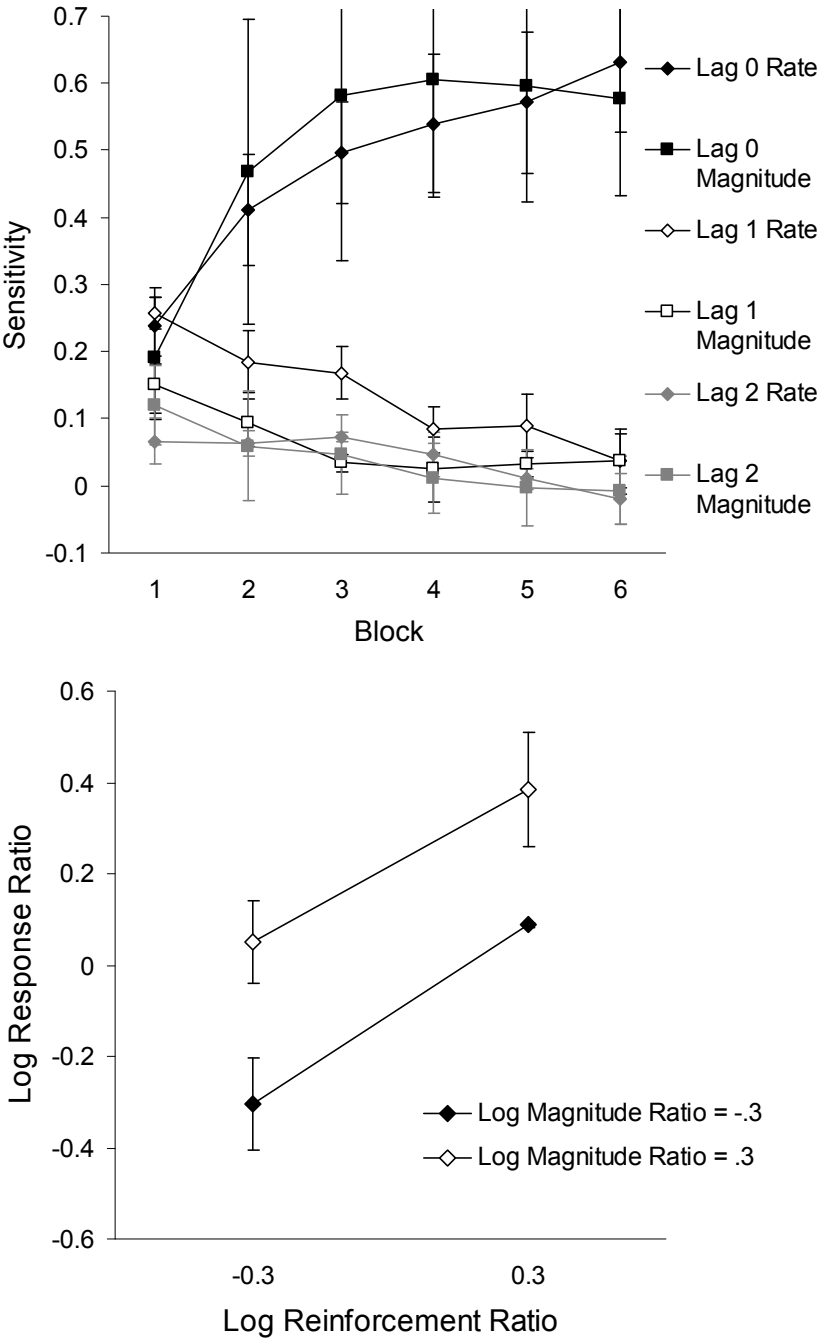

Figure 5.13. The top panel shows sensitivity to log reinforcement and magnitude ratios for Lag 0 through Lag 2 for each block of 12 interreinforcer intervals. Diamond data points represent reinforcement ratios and square data points represent magnitude ratios. The bottom panel shows mean log response ratio from the final block as a function of log reinforcement ratio when log magnitude ratio was positive (filled data points) and negative (unfilled). In both panels, bars represent standard error.

A concatenated generalized-matching model (Kyonka and Grace, 2008) provides a quantitative assessment of the degree of control over response allocation by the reinforcement and magnitude ratios from current and prior sessions:

$$\log\frac{B_{0L}}{B_{0R}} = a_{0R}\log\frac{R_{0L}}{R_{0R}} + a_{0M}\log\frac{M_{0L}}{M_{0R}} + a_{1R}\log\frac{R_{1L}}{R_{1R}} + a_{1M}\log\frac{M_{1L}}{M_{1R}} + a_{2R}\log\frac{R_{2L}}{R_{2R}} + a_{2M}\log\frac{M_{2L}}{M_{2R}} + \log b$$

Equation 5.9

In Equation 5.9, *B*s are numbers of key pecks, *R*s are reinforcer rates, *M*s are reinforcer magnitudes, *a*s are sensitivity coefficients, and log *b* is response bias. The subscripts *L* and *R* respectively refer to the left and right alternatives and numeric subscripts refer to session lag.

Multiple regressions were used to obtain parameter estimates for Equation 5.9 for each block of 12 interreinforcer intervals within each session for individual subjects. The upper panel of Figure 5.13 shows the mean sensitivity coefficients (values of *a*) for reinforcement and magnitude ratios from the current session (Lag 0) and two previous sessions (Lags 1 and 2), averaged across subjects. Lag 0 sensitivity coefficients for both dimensions increased over the course of the session and were statistically significant and large compared to those for higher lags, which did not change systematically or decreased. Results of regressions for individual subjects were similar to those shown in Figure 5.13. Average Lag 0 sensitivities to rate and magnitude (respectively) were 0.47 [SD = 0.15] and 0.49 [SD = 0.34] for responding over the whole session and 0.57 [SD = 0.21] and 0.59 [SD = 0.33] for responding in the second half. Table 5.6 shows Lag 0-2 rate and magnitude sensitivity coefficients for responding in the second half of sessions for individual subjects. Whereas Pigeons 195, 196 and 197 were more sensitive to rate than magnitude, Pigeon 198

was more sensitive to magnitude than rate. However, for all subjects, response allocation was a functional relation of Lag 0 rate and magnitude. Excepting Lag 1 rate for Pigeon 197, sensitivity coefficients for higher-Lag rates or magnitudes were not significant.

Multiple regression results confirm that both rate and magnitude ratios determine response allocation aggregated across sessions. The lower panel of Figure 5.13 shows the mean log response ratio from the final block of each session for each configuration of rate and magnitude ratios. The lines connecting data points with the same magnitude ratios appear parallel, which suggests that rate of reinforcement did not affect the relative effect of magnitude; that their additive effects are also independent. A repeated-measures analysis of variance (ANOVA) on mean response allocation from the final block with programmed Lag 0 log reinforcement and log magnitude ratios as factors verified main effects of rate, $F(1,3) = 35.01$, $p < .01$, $\eta_p^2 = 0.92$, and magnitude, $F(1,3) = 16.05$, $p < .05$, $\eta_p^2 = 0.84$. The interaction between rate and magnitude was not significant, $F(1,3) = 0.31$, *ns*, $\eta_p^2 = 0.09$.

Multiple regression and ANOVA results suggest the concatenated generalized matching law describes response allocation aggregated across sessions: Lag 0 reinforcement and magnitude ratios exert additive, independent control over response allocation, which increases over the course of the session. Two possible explanations are consistent with these results but make different predictions about response allocation within individual sessions: If concatenated matching describes response allocation in individual sessions, both reinforcer dimensions should determine response allocation in every session, and therefore it should be less extreme in tradeoff sessions than in dominated sessions. However, multiple regression and ANOVA results are also consistent with the possibility that a single reinforcer dimension determines response allocation in individual sessions but

the controlling dimension changes across sessions. If that is the case, the valence of the other dimension should have no effect and there should be no systematic difference between levels of preference in tradeoff and dominated sessions.

A measure of response allocation from the ends of sessions that was both stable and adequately representative was needed to distinguish between these explanations to eliminate the possibility that response allocation in tradeoff sessions was closer to indifference than in dominated sessions because preference took longer to stabilize and not because it was less extreme once it did stabilize. Examination of cumulative-response scatterplots (cf. Gallistel et al., 2001), which plotted the number of right responses as a function of the number of left responses for each reinforcer delivery, revealed that changes in response allocation within sessions tended to be abrupt when they occurred at all. To analyze these changes systematically, parameter estimates were obtained for linear and bi-linear models that predicted right responses using left responses. *F* ratios determined whether the increased variance accounted for by the bi-linear over the linear model was significant ($p < .05$). Three sessions (1.2%) had multiple visually identifiable change points. The incremental variance was not significant for 16.3% of sessions and they were determined to have no changes in response allocation. Incremental variance was significant for 82.5% of sessions; these were determined to have a single change point, which was estimated using linear interpolation (for a detailed explanation of this procedure, see Kyonka and Grace, 2008). Log response allocation from the final block of 12 interreinforcer intervals was considered a suitable measure of final, stable preference, provided no change point occurred during that period. On this basis, the three sessions with multiple change points and six additional sessions with change points that occurred during the final block were excluded from the analyses described below.

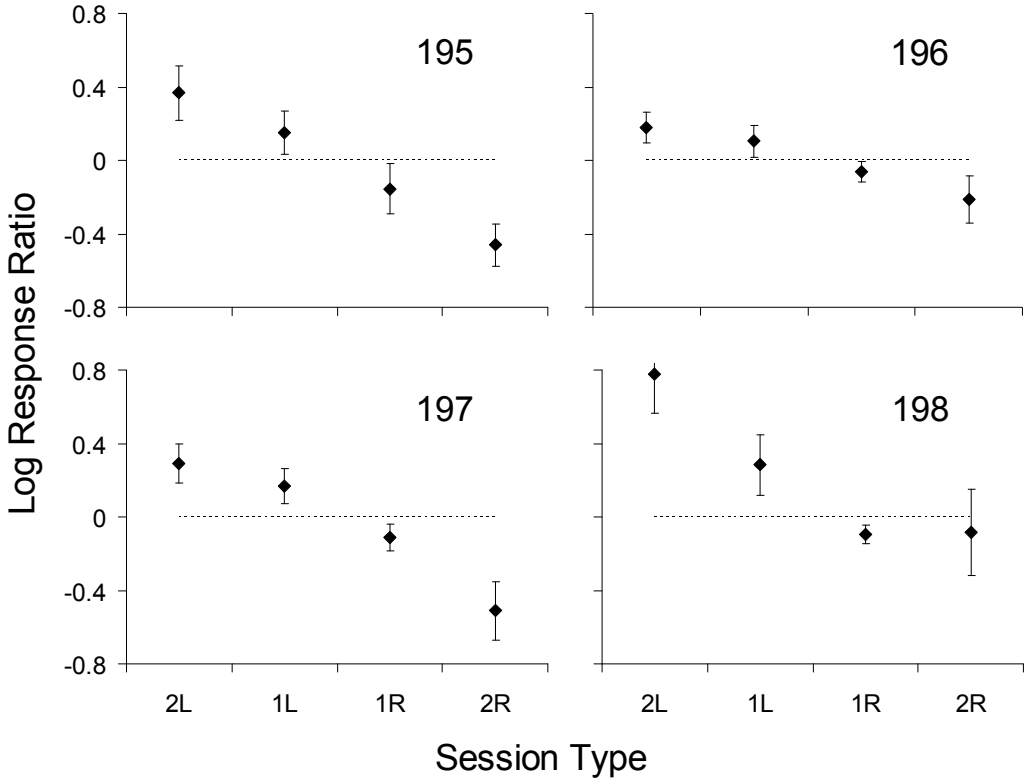

Figure 5.14. For each subject, mean log response ratios from the final block according to session type (see text for further explanation). Dashed lines show indifference – an equal number of responses to each alternative. Error bars show standard deviation.

Figure 5.14 shows mean log response ratios from the final block for each subject. Error bars show standard deviation instead of standard error as in the lower panel of Figure 5.13 because Figure 5.14 shows individual data. The sessions were grouped according to the pigeons' preferred alternative and the number of reinforcer dimensions favoring that alternative. Grouping sessions in this manner rather than according to configuration ensured that intermediate levels of preference in tradeoff sessions could not be attributable to inconsistent preference – a combination of extreme preference for the left alternative in some sessions with a particular configuration and extreme preference for the right in others. Left-dominated sessions (i.e., both richer rate and larger magnitude were associated with the left alternative) in which response allocation favored the left were labeled "2L". Tradeoff sessions were "1L" and "1R" when response allocation favored the left and right

alternatives, respectively. Any tradeoff session in which log response allocation was greater than zero was labeled "1L," regardless which reinforcer dimension favored which alternative. A combination of tradeoff session configurations comprised both "1L" and "1R" sessions for all subjects. Right-dominated sessions in which response allocation favored the right were labeled "2R". Pigeon 198 had a left-key bias and made more responses to the left alternative in four right-dominated sessions; these were not included in Figure 5.14. With one exception, response allocation in tradeoff sessions was less extreme than response allocation in dominated sessions, which suggests that both rate and magnitude determined response allocation in individual sessions.

If a single reinforcer dimension determined preference but the controlling dimension was resampled after every reinforcer, the intermediate preference obtained in tradeoff sessions would have to be a combination of extreme preferences for different alternatives. Figure 5.15 shows relative frequency distributions of choice proportions for individual interreinforcer intervals, binned in five intervals of width = .20, for each subject and type of session. Choice proportions from the 61st to 72nd intervals of the same sessions included in Figure 5.14, sorted in the same manner, comprise the distributions. If preference was always extreme within individual intervals, then distributions of choice proportions in tradeoff sessions should show evidence of bimodality. Instead, distributions appear unimodal. To confirm quantitatively that the observed distributions in tradeoff sessions could not be described by extreme preference on individual intervals, a Kolmogorov-Smirnov test compared observed distributions to those predicted if the choices in individual interreinforcer intervals were constrained to be in the extreme bins (0.0 to 0.2, and 0.8 to 1.0), but yielded the same overall choice proportion as that obtained. Kolmogorov-Smirnov tests showed that the obtained distributions were significantly

different from the simulated distributions (all *p*s < .001). This suggests that the intermediate preferences in tradeoff sessions were not produced by shifts in the controlling dimension across the last block. Instead, results are consistent with the prediction of the concatenated matching law that response allocation is determined by multiple reinforcer variables.

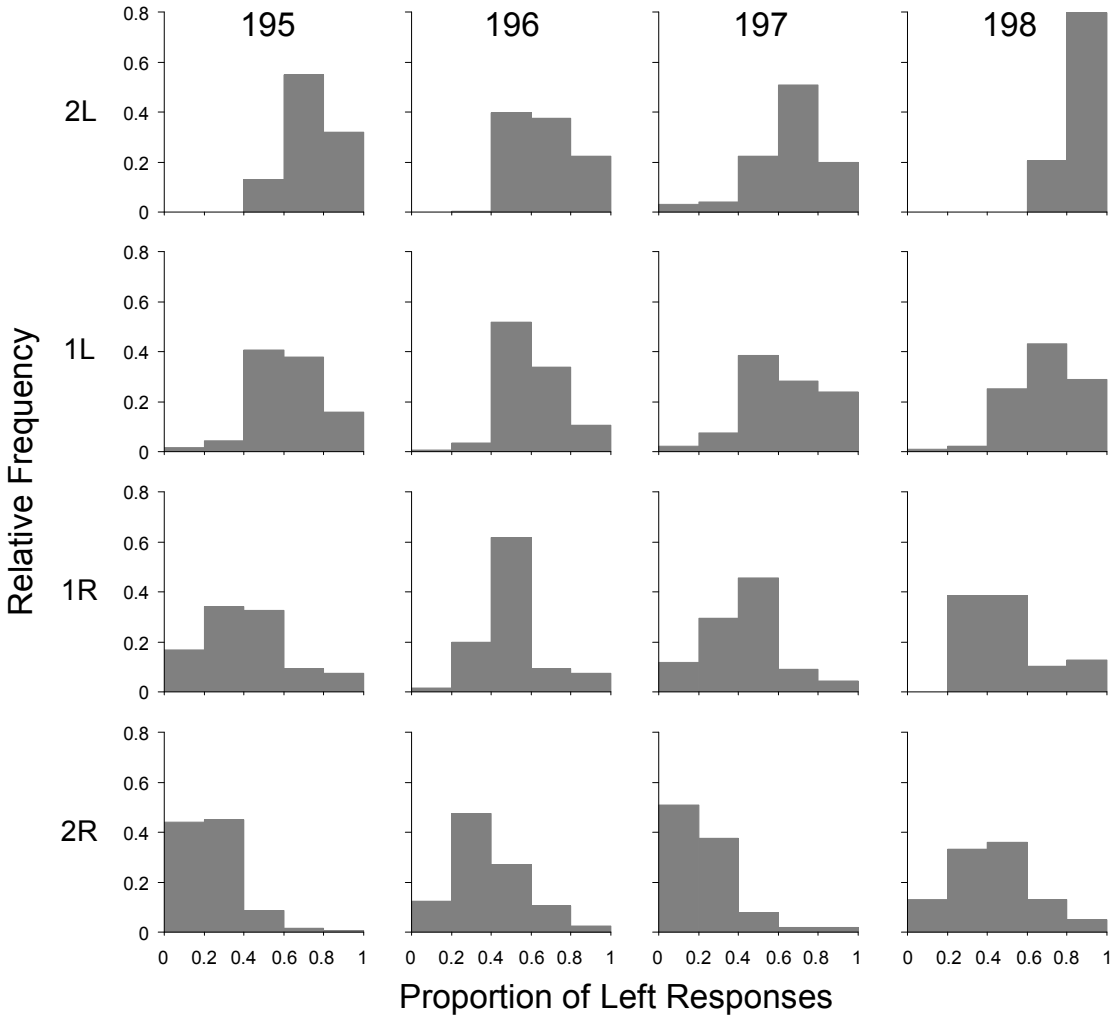

Figure 5.15. Relative frequency distributions of the proportion of left responses from each of the 61st through 72nd trials of all sessions included in Figure 5.14, for each subject and type of session.

5.7.5 Discussion

In this concurrent VI-VI schedule procedure, reinforcement ratios and reinforcer magnitude ratios were each either 2:1 or 1:2, determined across sessions by independent random series. Consistent with the concatenated generalized matching law's assumptions of additivity and independence, log reinforcement and log reinforcer magnitude ratios from the current session controlled response allocation aggregated across sessions, but their interaction did not. Response allocation was less extreme in tradeoff sessions, in which rate favored one alternative and magnitude the other, than in dominated sessions in which both dimensions favored the same alternative. This suggests that both rate and magnitude determined pigeons' responding in individual sessions. The intermediate levels of preference obtained in tradeoff sessions were not a combination of extreme preferences for different alternatives in individual interreinforcer intervals: distributions of single-interval choice proportions were unimodal. The results of this study are qualitatively similar to those obtained by Kyonka and Grace (2008) in concurrent-chains experiments with multiple frequently-changing reinforcer dimensions. This suggests that a similar process underlies performance in concurrent schedules and concurrent chains in which reinforcement contingencies change unpredictably across sessions.

Two concurrent-schedule studies in which contingencies were determined each session according to pseudorandom binary sequences report sensitivity coefficients from log response allocation regressed on log reinforcement ratios or log reinforcer magnitude ratios from current and previous sessions. For the two conditions in which reinforcer ratios were 2:1 or 1:2, Schofield and Davison (1997) reported an average sensitivity to Lag 0 log reinforcement ratio of 0.72, greater than that obtained in the present experiment but with considerable overlap in the ranges of sensitivity coefficients for individual subjects.

Maguire et al. (2007) reported an average sensitivity to Lag 0 log magnitude ratios of 0.14, lower than that obtained in the present experiment. When the reinforcer is access to food provided via the presentation of a grain magazine, reinforcer magnitude can be manipulated by varying the duration of access or the number of successive magazine presentations. Whereas the present experiment varied duration, Maguire et al. (2007) varied number of presentations in ratios of 1:7 and 7:1. Using a different but related procedure, Davison and Baum (2003) manipulated reinforcer magnitude across components or across conditions and found that sensitivity to magnitude ratios was lower than sensitivity to rate, consistent with steady-state findings. However, Landon et al. (2003) obtained strong control by magnitude in a concurrent schedule procedure in which the number of successive presentations for each reinforcer changed several times each session, so there is little evidence that pigeons are more sensitive to differences in duration of access than to number of hopper presentations. Lower sensitivity to a 7:1 ratio than a 2:1 ratio could be consistent with a ceiling effect or a nonlinear relationship between log response allocation and log reinforcer magnitude (c.f. Davison & Jones, 1995). Future work investigating possible nonlinear relationships between response allocation and reinforcer magnitude in an unpredictably-changing choice procedure will require more than two different reinforcer magnitude ratios.

Although sensitivity to rate was lower and sensitivity to magnitude higher than values reported in previous studies, sensitivity coefficients for both reinforcer dimensions manipulated in this experiment were similar to each other and within the range of sensitivities reported in concurrent schedule experiments in which a single reinforcer dimension changed. Kyonka and Grace (2008) found that increasing the number of dimensions that changed unpredictably did not decrease sensitivity to any single dimension.

Taken together, these results suggest that different reinforcer dimensions do not compete to control response allocation – perhaps a surprising result given the importance of limited attentional capacity to multiple-cue probability learning (Young and Wasserman, 2002), but consistent with the concatenated generalized matching law's assumptions that the effects of multiple reinforcer dimensions on preference are additive and independent.

6 Chapter 6 All that we have to decide is what to do with the time that is given us

The present research includes five concurrent-chains experiments and one concurrent-schedule experiment in which the contingencies that determine reinforcement on left and right keys change unpredictably across sessions. The primary objective of this research was to identify the process that generates choice between delayed food rewards and to characterize its relationship with temporal control. This chapter is an attempt to summarize and integrate results from all six experiments with respect to the theoretical assumptions made by accounts of choice derived from cognitive theories of timing (Gallistel & Gibbon, 2000; Gibbon, 1977) and those based on conditioned reinforcement (Fantino, 1969; Grace, 1994; Mazur, 2001). To provide a quantitative account of the data, a quasidynamic decision model for choice between delayed rewards (Grace & McLean, 2006) is applied to Experiments 1-3 and a modified and extended version is applied to Experiments 4-6. Although there are regularities in initial-link performance that the decision model does not capture, notably the abrupt nature of changes in response allocation within individual sessions, it can nevertheless describe the major features of concurrent choice behavior.

This final chapter begins with attempts to provide integrative summaries of the major empirical results of the present research. The dynamics of choice and timing, and the nature of quantitative relationships between measures of choice and timing in Experiments 1-3, are presented in Section 6.1. Section 6.2 is a discussion of the effects of multiple unpredictably changing reinforcer dimensions in Experiments 4-6. Following the empirical summaries, in Sections 6.3 and 6.4 versions of Grace and McLean's (2006) decision model

for delayed rewards and for expectancy are respectively applied to Experiments 1-3 and Experiments 4-6. The final section includes an evaluation of different possible mechanisms for generating response allocation and timing in concurrent choice procedures.

6.1 Rapid acquisition of choice and timing

Although there has been much empirical and theoretical work on choice and timing in the experimental analysis of behavior, there have been few attempts at integration. Gallistel, Gibbon and colleagues have applied Scalar Expectancy Theory (Gibbon et al., 1988) and Rate Estimation Theory (Gallistel & Gibbon, 2000) – cognitive accounts of timing relying on the construct of reward expectancy – to choice procedures including concurrent chains. Both theories are able to predict the major empirical phenomena in concurrent chains experiments, including matching and overmatching of initial-link response allocation to relative immediacy with VI and FI terminal links, respectively. However, both theories assume that subjects sample delays for each alternative from distributions with scalar variability and that responding reflects a winner-take-all preference for the alternative associated with the shorter delay; assumptions which were heretofore largely untested (c.f. Brunner et al., 2000). Experiments 1-3 were attempts to lay an empirical foundation for an integrated study of choice and temporal control in which within-session acquisition of both was investigated. These experiments involved concurrent-chains procedures in which some terminal links were not reinforced, analogous to no-food trials in the peak procedure. In initial links, pigeons chose between terminal-link stimuli that were associated with different FI schedules that changed unpredictably across sessions. Measures of temporal control were obtained from responding on no-food terminal

links. Thus, the procedures provided convergent measures of choice and timing under dynamic conditions.

The position of the shorter terminal link varied randomly from left to right across sessions according to pseudorandom binary sequences. There were two types of conditions. Experiment 2 and one condition of Experiment 1 were 'minimal variation', in which terminal-link schedules were FI 10 and 20 s. Experiment 3 and the other condition in Experiment 1 were 'maximal variation', in which terminal-link schedules were sampled from a potentially infinite population of values. There were three major objectives of these studies. One objective was to characterize within-session acquisition trajectories of initial-link response allocation, and start and stop times (measures of temporal control from no-food trials). A second was to identify the relationship between choice and timing acquisition. A third objective was to test whether initial-link choice was determined by an opting, categorical decision process, as proposed by Gallistel and Gibbon (2000), or by matching to relative value as proposed by contemporary theories of concurrent-chains choice (Grace, 1994; Mazur, 2001).

6.1.1 Lag 0 immediacy ratios govern initial-link response allocation

For all subjects in all conditions of Experiments 1-3, initial-link response allocation adjusted to unpredictable changes in terminal-link delay and stabilized within-session, on average, halfway through sessions of 72 cycles' duration. There was no systematic effect of immediacy ratios from previous sessions on response allocation. Acquisition of preference was rapid, with response allocation stabilizing in minimal- and maximal variation conditions of all experiments about midway through the sessions. Aggregated across sessions, sensitivity to immediacy ratios in the current session (i.e., Lag

0) increased gradually over the first half of sessions. However, analysis of cumulative response plots from individual sessions of Experiment 1 showed that changes in response allocation tended to be abrupt.

Experiments 2 and 3 were attempts to characterize effects of absolute initial- and terminal-link duration when immediacy ratios changed unpredictably across sessions. In Experiment 2, the programmed VI schedule value of initial links was sampled each session from a uniform distribution. Response allocation was a monotonically decreasing function of initial-link duration. In contrast to results of a rapid acquisition experiment in which initial-link duration changed systematically across sessions according to an ascending and descending series (Christensen & Grace, 2008), there was no evidence of nonlinearity in the relationship between preference and initial-link duration. In Experiment 3, pairs of terminal-link delays either summed to 15 s or to 45 s each session and PI schedule values changed unpredictably across sessions. For all four pigeons, preference for the shorter terminal link was greater when terminal-link delays were short than when they were long. Thus, to a first approximation, effects of absolute initial- and terminal-link duration in Experiments 2 and 3 were comparable to those reported in steady-state research (Fantino, 1969; MacEwen, 1972; Williams & Fantino, 1978).

In maximal-variation conditions, it was impossible to predict the particular terminal-link delays that would be in effect at the beginning of a session. Subjects had no recent exposure to either delay. Therefore, determining which initial-link key produced the shorter terminal-link delay was more complicated in maximal-variation than in minimal-variation conditions. In Experiment 1, pigeons' sensitivity to relative immediacy was greater in the minimal- than the maximal-variation condition, but there was no evidence that acquisition was faster in the minimal- than in the maximal-variation condition. The

functional relationship between log initial-link response ratios and log immediacy ratios was not linear in maximal-variation conditions. Instead, for all subjects in Experiments 1 and 3, response allocation in the maximal-variation condition was a nonlinear function of the log terminal-link immediacy ratio. Data fell into two clusters depending on whether the left or right alternative was favored, consistent with a categorical discrimination. Although subjects preferred the shorter delay reasonably consistently, the probability that response allocation would favor the longer alternative was greater if delays were similar, resulting in a discontinuous relationship between response allocation and relative immediacy. This provides evidence that supports the opting process assumption of RET (Gallistel & Gibbon, 2000) and challenges the correspondence between preference and relative value assumed by matching-inspired models (Grace, 1994; Mazur, 2001).

6.1.2 Lag 0 terminal-link delays govern start and stop times

Similar to initial-link response ratios, start and stop times from no-food terminal links in Experiments 1-3 adjusted to unpredictable changes in terminal-link delays, stabilizing at or before the midway point in the session. However, there were a number of differences between control of log initial-link response ratios by log terminal-link immediacy ratios and control of start and stop times by terminal-link delays. Start and stop times were linear functions of terminal-link schedule values in maximal-variation conditions. Initial-link duration had no effect on start or stop times. Absolute terminal-link duration had no effect on temporal control of stop times. Sensitivity to schedule value was greater with long than short delays for two subjects in Experiment 3, but not systematically different for the other two subjects.

Variability of start- and stop-time responding in steady-state peak procedure experiments is scalar, that is, a constant proportion of the schedule value (Church, Meck & Gibbon, 1994; Crystal, Church & Broadbent, 1997). Stop times were linearly related to delay, comparable to stop times obtained in steady-state peak procedure research, acquired within session, and consistent across subjects and Experiments. Therefore, stop time can be considered a reliable measure of temporal control in rapid acquisition choice procedures. Start times were linearly related to delay and acquired within session, but violated principles of timescale invariance and the scalar property in some instances, unlike stop times. Moreover, relative variability of start times was always greater than relative variability for stop times. Taken together, these results suggest that start times were a less reliable measure of temporal control in these procedures than stop times.

6.1.3 Covariation of choice and timing

Patterns of covariation were examined to determine the relationship between temporal control and choice in Experiment 1. According to cognitive accounts of choice (Gallistel & Gibbon, 2001; Gibbon et al., 1988), a common representation of delay in memory determines both choice and timing behavior. If there was a systematic relationship between measures of temporal control and of choice in concurrent chains, initial-link response strength and terminal-link start and stop times should be determined according to some formulation of the following:

$$DV_i = f\left(D_i + \varepsilon_{REPi}\right) + \varepsilon .$$

Equation 6.1

In Equation 6.1, DV_i refers to the dependent variable being measured (i.e., log response ratio, start time, stop time or, theoretically, any other measure of choice or timing). There

are two types of error: unsystematic error, ε, and 'learning' error, ε_{REPi}, which is common to initial- and terminal-link measures of choice and timing behavior. The function $f(D_i + \varepsilon_{REPi})$ could be an arbitrary transformation (Killeen, 1972) of terminal-link delay plus learning error, and need not be the same for start, stop and initial-link response strength. In other words, learning error should produce additional covariation of relative start and stop times with initial-link response allocation, controlling for respective relationships with relative terminal-link delay.

Because initial-link response allocation is a dimensionless, relative measure, it was necessary to compare log initial-link response ratios with log ratios of start and stop times, i.e., log[StartR/StartL] and log[StopR/StopL]. All measures should be positively correlated with log immediacy ratio (L/R), therefore additional covariation between measures of choice and temporal control would appear as positive partial correlations and between log response and log stop time ratios, controlling for effects of log immediacy ratio. In Experiment 1, partial correlations between log response and log start time ratios, calculated for each subject over single session blocks (six per session) and then pooled across subjects, were always positive and statistically significant for all blocks of the maximal-variation condition and for all but Blocks 2 and 3 of the minimal-variation condition. Partial correlations between log response and log stop time ratios were positive and statistically significant for all blocks of both conditions.

Table 6.1. Analysis of residual correlations, Experiment 2. For Start Time and Stop Time analyses, respectively, partial correlations were computed between log initial-link response ratios and log start time ratios and between log initial link response ratios and log stop time ratios, controlling for log immediacy ratios.
*** p < 0.05, ** p < 0.01, *** p < 0.001**

	Block					
	Start Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
111	0.29*	0.23	0.28*	0.13	0.27	-0.26
112	0.05	0.51***	0.58***	0.15	0.60***	0.35*
113	0.17	0.38*	0.57***	0.36*	0.29*	-0.04
114	0.02	0.28*	-0.03	-0.01	0.36*	-0.24
Pooled	0.14	0.35***	0.48***	0.26***	0.34***	0.00
	Stop Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
111	0.06	0.21	0.04	-0.13	-0.15	-0.05
112	0.17	0.37**	0.17	0.04	-0.02	0.21
113	0.17	0.51***	0.57***	0.37**	0.21	0.31
114	0.21	0.14	0.24	-0.10	0.31*	-0.24
Pooled	0.14	0.40***	0.42***	0.16*	0.11	0.16*

Table 6.2. Analysis of residual correlations calculated from Short sessions of Experiment 3. For Start Time and Stop Time analyses, respectively, partial correlations were computed between log initial-link response ratios and log start time ratios and between log initial link response ratios and log stop time ratios, controlling for log immediacy ratios.
*** p < 0.05, ** p < 0.01, *** p < 0.001**

	Block					
	Start Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
221	-0.22	0.17	0.26	0.18	-0.03	0.16
222	-0.47**	-0.35*	-0.18	0.06	0.08	0.09
223	0.41**	0.09	0.31*	0.28*	0.26	0.37**
224	-0.15	-0.28	0.18	-0.09	-0.06	0.08
Pooled	-0.03	-0.11	0.19**	0.08	0.06	0.17*
	Stop Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
221	0.46**	0.27	0.18	0.38**	0.15	-0.01
222	0.36*	0.30*	0.22	0.29*	0.50***	0.27
223	0.42**	0.47**	0.13	0.22	0.17	0.23
224	0.56***	0.43**	0.05	0.20	0.45**	0.28
Pooled	0.44***	0.36***	0.12	0.24**	0.34***	0.22**

Table 6.3. Analysis of residual correlations calculated from Long sessions of Experiment 3. For Start Time and Stop Time analyses, respectively, partial correlations were computed between log initial-link response ratios and log start time ratios and between log initial link response ratios and log stop time ratios, controlling for log immediacy ratios.
*** p < 0.05, ** p < 0.01, *** p < 0.001**

	Block					
	Start Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
221	-0.01	0.41**	0.42**	0.50***	0.19	0.06
222	-0.33*	0.10	0.07	0.03	0.01	0.18
223	0.01	0.05	0.03	-0.06	0.08	0.28
224	-0.06	0.30*	0.20	0.44**	0.11	0.41**
Pooled	-0.10	0.22**	0.18*	0.24**	0.09	0.22**
	Stop Time					
Pigeon	1st	2nd	3rd	4th	5th	6th
221	-0.04	0.06	0.28	0.40**	0.24	0.28
222	0.52**	0.13	0.54***	0.39*	0.52**	0.42**
223	0.29	0.37*	0.26	0.14	0.41**	0.23
224	0.08	0.32	0.42*	0.09	0.56**	-0.03
Pooled	0.20**	0.21**	0.37***	0.23**	0.42***	0.22**

Table 6.1 shows results of residual covariation analyses for Experiment 2. Tables 6.2 and 6.3 show results from the same analyses for Short and Long sessions from Experiment 3, respectively. Of 72 (6 blocks x 4 subjects in Experiment 2 and 6 blocks x 4 subjects x 2 types of session in Experiment 3) individual start time partial correlations, 54 were positive and 22 of those were statistically significant. There were 63 positive individual stop time partial correlations, 26 significant. Of the 18 negative individual start time partial correlations, only 3 were significant and none of the 9 negative individual stop time partial correlations were significant. With few exceptions, correlations based on data pooled across subjects were positive and statistically significant. Thus, residual covariation of relative start and stop times with initial-link response allocation in Experiments 2 and 3 was broadly consistent with that reported in Experiment 1. That residuals of log start time ratio are less reliably correlated with those of log response ratios than log stop time ratio

residuals across experiments is consistent with the finding that start times are relatively more influenced by nontemporal factors than stop times.

6.2 Rapid acquisition and multiple reinforcer dimensions

In Experiments 4-6, the left:right position of the richer rate, larger magnitude and greater probability of reinforcement changed unpredictably across sessions, in addition to or instead of that of the shorter terminal-link delay. Reinforcer ratios (immediacy and magnitude in Experiment 4, immediacy, magnitude and probability in Experiment 5 and rate and magnitude in Experiment 6) were 2:1 or 1:2 in each session, and changed across sessions according to independent, random sequences. The objectives of these studies were to determine whether multiple unpredictably changing reinforcer dimensions could affect pigeons' response allocation in free-operant choice procedures, concurrently or otherwise, and to distinguish between two possible underlying processes that might produce sensitivity to multiple reinforcer dimensions at the molar level. One of these possibilities was that, within any temporal epoch, ratios of different reinforcer dimensions combine additively and independently (in logarithmic terms) to determine response allocation. The other was that only one reinforcer ratio determines response allocation within a temporal epoch, but the controlling dimension changes across epochs.

6.2.1 Lag 0 reinforcer ratios govern response allocation at the molar level

Across sessions, log response ratios from Experiment 6 and initial links of Experiments 4 and 5 were sensitive to all reinforcer dimensions. As in Experiments 1-3, there was no systematic effect of reinforcer ratios from previous sessions. On average, response allocation stabilized about midway through sessions. Analysis of cumulative response plots from individual sessions of Experiments 4 and 5 showed that changes in

response allocation tended to be abrupt. However, at the molar level, responding reflected an additive combination of log reinforcer ratios of multiple dimensions; across sessions, preference was less extreme when at least one dimension favored each key than when all dimensions favored the same key.

An assumption of concatenated-matching analyses of response allocation (Equation 1.7) is that effects of ratios from different reinforcer dimensions are additive and independent in logarithmic terms. This was so in Experiments 4 and 6: interactions between immediacy and magnitude and rate and magnitude typically were not significant for individual subjects or consistent across subjects. Similarly, two-way interactions in Experiment 5 were neither significant nor systematically positive or negative. However, three-way immediacy x magnitude x probability ratio interactions were statistically significant and consistent across subjects: the effect of any one dimension was greater when the other two dimensions favored different keys than when they favored the same key. The three-way interactions may indicate a ceiling effect on preference in rapid-acquisition concurrent chains with dependently scheduled initial links, rather than a violation of independence of reinforcer dimensions.

6.2.2 Response allocation within temporal epochs

To distinguish between the possibilities listed at the beginning of Section 6.2 using session as the temporal epoch, it was necessary to examine the level of preference from the final block of each session, after responding had stabilized. Within individual sessions, as at the molar level, preference was less extreme when at least one dimension favored each key than when all dimensions favored the same key in Experiments 4-6, contrary to the hypothesis that that pigeons' preference was determined by a single dimension sampled

from all differing dimensions. If pigeons were responding on the basis of a single reinforcer dimension, but the determining dimension was resampled after each reinforcer, responding should have been extreme in individual initial-link cycles of Experiments 4 and 5 and IFIs of Experiment 6. In this case, distributions of choice proportions from single initial links or IFIs would be bimodal; comprised of combinations of exclusive or extreme preference for one key or the other. Instead, all distributions were unimodal. Responding in smaller temporal epochs such as visits or single responses is constrained to be exclusive by the nature of the response (i.e., single pecks must be allocated entirely to one key or the other). Therefore, over the smallest temporal epoch at which it was possible to distinguish between winner-take-all sampling and matching to relative value, multiple reinforcer ratios determined pigeons' responding. It appears that Lag 0 reinforcer ratios (rate, immediacy, magnitude and probability) govern response allocation, within individual temporal epochs as well as at the molar level.

6.3 Applying the decision model to choice between delayed rewards

In the six Experiments presented in this thesis, log immediacy, magnitude, probability and/or reinforcer ratios governed pigeons' response allocation. At the molar level, sensitivity to these log ratios increased over the first three blocks of 12 cycles and stabilized thereafter. Can a single model account for both effects of log ratios and changes in sensitivity over the course of sessions? The decision model proposed by Grace and McLean (2006) predicts response allocation based on relative expected response strength, which is determined in Grace and McLean's model by the relative probabilities that left and right terminal links are categorized 'short' in comparison with a criterion. The decision model describes average log response ratio within sessions as a function of experience with

terminal-link delays and across sessions as a function of log immediacy ratios. It has been applied to initial-link performance in rapid-acquisition concurrent chains in which terminal-link schedules changed systematically (Christensen & Grace, 2009a) and unsystematically (Christensen & Grace, 2008; Grace & McLean, 2006). It should be able to account for effects of relative and absolute terminal-link duration, and absolute initial-link duration for concurrent chains in which log immediacy ratios changed pseudorandomly across sessions. The purpose of this section is to fit the decision model to initial-link performance in Experiments 1-3.

According to the decision model, each time a subject experiences a terminal link, that terminal-link delay is compared with a criterion. Response strength increases if the delay is categorized 'short' and decreases if it is categorized 'long' relative to the criterion. Expected response strength increases in proportion to the probability the delay is 'short' and decreases in proportion to the probability it is 'long'. Response allocation and Lag 1 reinforcer ratios were never correlated in Experiment 1 or other unpredictably-changing rapid acquisition procedures (Grace, Bragason & McLean, 2003; Schofield & Davison, 1997); pigeons approached each session of Experiment 1 *de novo*. For this reason, the decision models applied in this section and the next assume that response strength prior the beginning of each session, RS_0, always resets to the same default value, the geometric mean of the minimum and maximum possible response strengths. For Experiments 1-6, minimum and maximum response strengths were always $Min_{RS} = 0.01$ and $Max_{RS} = 1$, respectively, so RS_0 was 0.10. Over the course of each session, a linear operator adjusts response strength according the probability the just-experienced delay was categorized short:

$$RS_{n+1} = RS_n + p_{short} * (Max_{RS} - RS_n) * \Delta - (1 - p_{short}) * (RS_n - Min_{RS}) * \Delta .$$

Equation 6.2

The Δ parameter limits rate of acquisition. The decision model assumes that the probability a delay would be categorized short, p_{short}, equals the probability that the log terminal link delay was less than a randomly-selected log delay from a normal distribution:

$$p_{short} = 1 - \Phi(\log D, \log C, \sigma) .$$

Equation 6.3

In Equation 6.3, Φ is the proportion of a cumulative normal distribution with mean log*C* and standard deviation σ between -∞ and log*D*. *D* is terminal-link delay to food on reinforced trials. In other words, p_{short} is the proportion of a normal distribution (with mean log*C* and standard deviation σ) that is greater than log*D*. In the initial application of the decision model, Grace and McLean (2006) assumed that log*C* was the log geometric mean of the terminal-link delays.

To account for systematic changes in absolute initial- and terminal-link duration, Christensen and Grace (2008) proposed an extension to the decision model. They assumed that the criterion was determined by the intervals between all stimuli correlated with reinforcement, including both initial-link onset to terminal-link entry, and terminal-link entry to reinforcement. Suppose that the criterion mean, log *C*, is computed as an exponentially-weighted moving average[1] of the average log initial- and terminal-link delays from the current and previous sessions:

$$\log C_N = \sum_{i=0}^{N} \beta * (1 - \beta)^{N-i} * \left(\frac{[2 * \log IL_i + \log D_{Li} + \log D_{Ri}]}{4} \right), \text{ or equivalently,}$$

$$\log C_{N+1} = \beta * \left(\frac{\left[2 * \log IL_N + \log D_{LN} + \log D_{RN} \right]}{4} \right) + (1 - \beta) * \log C_N \ .$$

Equation 6.4

In Equation 6.4, IL_i, D_{Li} and D_{Ri} are initial-link, left terminal-link and right terminal-link delays in the *i*th (or *N*th) session. The log*IL* value is multiplied by 2 because initial links occur in each cycle, therefore the subject experiences them twice as often as either terminal-link delay. β is a value between 0 and 1. Larger β values indicate greater weight given to the current session as a determiner of the criterion, relative to previous sessions. If β is 1, the criterion is determined by Lag 0 delays exclusively. If β is 0, a single, unchanging criterion applies to performance across all sessions.

The decision model predicts an effect of initial-link duration because the calculation of average delay includes initial-link intervals. Christensen and Grace (2008) noted that the decision model (correctly) predicts that the effect is bitonic, and provided and explanation why that is the case: Log initial-link response ratio is positively correlated with the relative probabilities of 'short' decisions for left and right terminal links. Therefore, greatest sensitivity to log immediacy ratios occurs when p_{short} for the left terminal-link and p_{short} for the right terminal link are maximally different. Sensitivity is lower for short and long initial-link durations because probabilities of 'short' decisions for FI 10 s and FI 20 s terminal links are both low or both high, respectively.

[1] Darren Christensen developed the exponentially-weighted moving average criterion to account for differences in rapid acquisition and steady-state performance in his doctoral thesis (D. Christensen, personal communication, 2007).

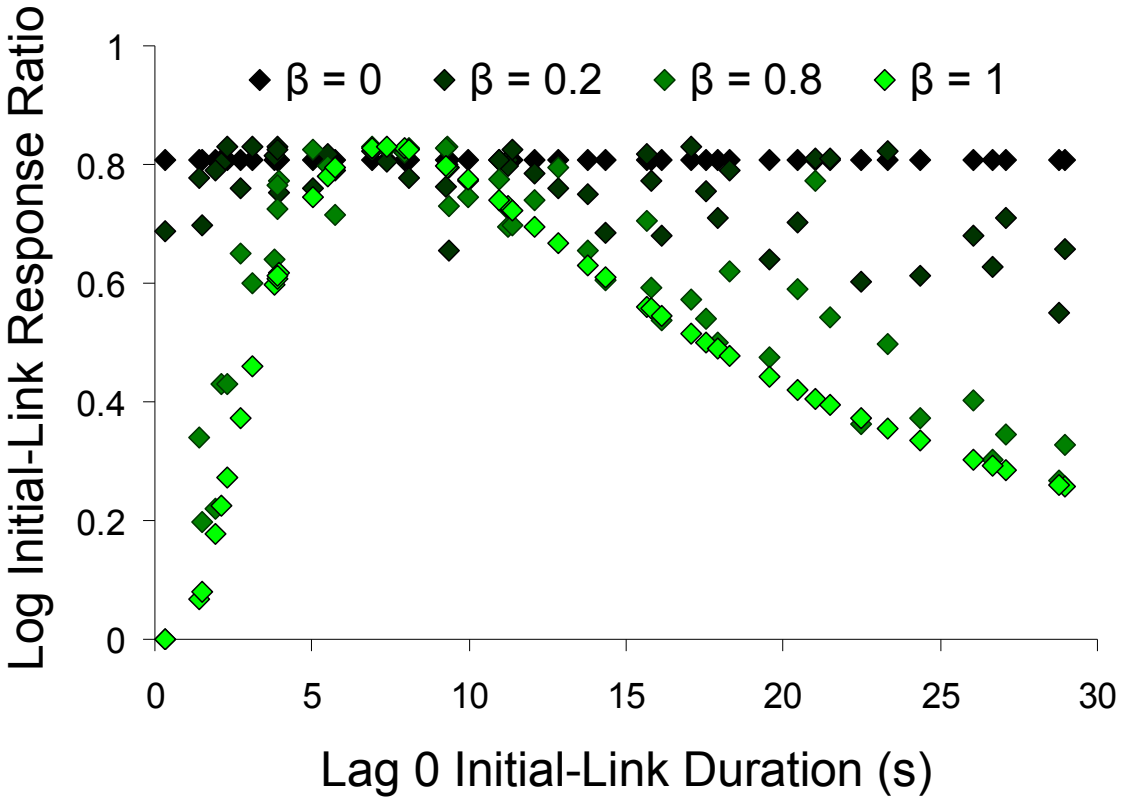

Figure 6.1. Preference for the shorter (FI 10 s) terminal link as predicted by the decision model using different values of β. The sequence of initial-link schedule values used was the pseudorandom sequence experienced by Pigeon 111 in Experiment 3. For all series, σ = 0.15 and Δ = 0.2.

Figure 6.1 shows the predicted effect of initial-link duration on preference for the shorter terminal link when initial-link duration changes pseudorandomly across sessions, for different values of β. When β is zero, there is no effect of initial-link duration, because the criterion does not change across sessions. Christensen and Grace (2008) reported a bitonic initial-link effect for $\beta > 0$. For $0 < \beta < 1$, initial- and terminal-link schedules from current and previous sessions determine the criterion. Lower-Lag (that is, more recent) sessions always have greater influence than higher-Lag sessions. When initial-link duration changed systematically across sessions in Christensen and Grace (2008), initial-link duration from recent sessions was always similar to initial-link duration in the current (Lag 0) session, thus log*C* for any Lag 0 initial-link duration was similar for all values of β except 0. However, when initial-link duration was changed pseudorandomly across

sessions in Experiment 3, initial-link duration from previous sessions could be much shorter or longer than Lag 0 initial-link duration, which would increase or decrease log*C* by an amount determined by β. The 'scatter' present in the β = 0.2 and β = 0.8 series of Figure 6.1 is attributable to the particular sequence of initial-link values used in the simulation. It is possible that, for certain values of β, the scatter from the sequence of initial-link durations would be large enough to obscure the bitonic element of the initial-link effect but not so large that it obscured the linear element of the initial-link effect. Hence, the decision model can account for performance in Christensen and Grace (2008) and Experiment 2.

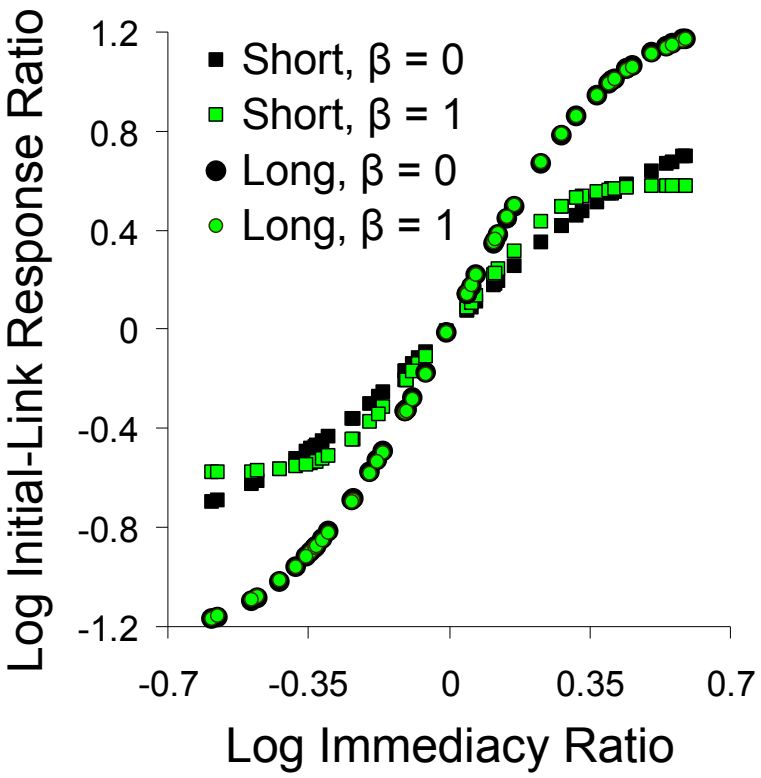

Figure 6.2. Log initial-link response allocation predicted by the decision model using β = 0 and β = 1 for Short and Long pairs of terminal-link delays (i.e., left and right terminal link delays sum to 15 and 45 s, respectively). For all series, σ = 0.15 and Δ = 0.2.

When the criterion changes across sessions according to Equation 6.4, the decision model predicts the terminal-link effect as well as the initial-link effect. This is because with the same initial-link schedules, logC does not increase proportionately with absolute

terminal-link duration. Figure 6.2 shows log initial-link response ratios plotted as a function of log immediacy ratio, predicted by the decision model using β values of 0 (in black) and 1 (in green). Separate predictions (using the same parameter values) were generated for Short pairs of delays that summed to 15 s and for Long pairs that summed to 45 s. Sensitivity to relative immediacy is greater for Long than Short delays for both β values.

Table 6.4. Decision model parameter estimates and variances accounted for (VACs) by the decision model and the generalized matching law (GML), based on performance blocks 4-6 of sessions from Experiments 1-3, using programmed initial- and terminal-link delays. Criterion mean (log*C*) was 1.08 in Experiment 1, 1.05, 1.06, 1.21 and 1.05 for Pigeons 111, 112, 113 and 114 in Experiment 2 and 0.92, 1.16 and 1.04 for Short, Long and Mixed conditions in Experiment 3. Subscripts min and max refer to minimal- and maximal-variation conditions, respectively.

			Decision Model				Generalized Matching	
Pigeon	σ	β *	Δ	Log*b*	VAC_{min}	VAC_{max}	VAC_{min}	VAC_{max}
				Experiment 1				
111	0.12	-	0.18	0.08	0.90	0.72	0.90	0.72
112	0.05	-	0.11	-0.06	0.97	0.80	0.98	0.80
115	0.10	-	0.20	-0.08	0.98	0.68	0.98	0.68
116	0.15	-	0.20	-0.05	0.94	0.69	0.94	0.69
117	0.17	-	0.20	-0.12	0.86	0.71	0.86	0.71
				Experiment 2				
111	0.14	0.13	0.25	0.08	0.95	-	0.94	-
112	0.18	0.18	0.14	0.03	0.86	-	0.84	-
113	0.12	0.36	0.20	0.17	0.78	-	0.77	-
114	0.35	0.66	0.07	0.04	0.24	-	0.23	-
				Experiment 3				
221	0.08	0.75	0.16	0.02	-	0.78	-	0.78
222	0.20	0	0.15	-0.45	-	0.53	-	0.44
223	0.07	1	0.11	0.29	-	0.63	-	0.63
224	0.10	0.90	0.21	-0.03	-	0.71	-	0.69

* In Experiment 1, the β parameter was assumed to equal 0.

Table 6.4 shows decision model parameter estimates for Experiments 1-3. Log *b* is a global bias parameter that was added to log response-strength ratios to generate predicted log initial-link response ratios. It represents a constant influence on response allocation that was independent of changes in response strength predicted by the decision

model. Table 6.4 also shows VAC by the decision model and by a generalized-matching model (Equation 1.7), based on performance in the second half of sessions. Sessions in Experiments 1-3 all consisted of 72 dependently-scheduled initial- and terminal-link cycles. In each block of 12 cycles, subjects experienced the same number of reinforcers in each terminal link (5). For this reason, decision model predictions were generated for temporal epochs of 12-cycle blocks for the last 50 sessions of each condition except the final condition from Experiment 3, which was a replication. Parameters were estimated to maximize VAC across individual blocks. Decision model predictions for initial-link response ratio over the second half of sessions were based on the arithmetic mean left and right response strengths from blocks 4-6.

Decision model and generalized-matching VAC were similar for obtained log response ratios from blocks 4-6 in minimal- and maximal-variation conditions of Experiments 1-3. Does this imply that the decision model does not improve on generalized matching or theoretical models related to it? Is there anything the decision model can account for that generalized matching and steady-state theories of concurrent-chains choice cannot? The decision model can account for the monotonically-decreasing preference for the shorter, FI 10-s delay for longer initial links, but so can DRT (Fantino, 1969; Squires & Fantino, 1971), CCM (Grace, 1994) and HVA (Mazur, 2001). All four models correctly predict that the sensitivity of pigeons' log initial-link response ratios to log immediacy ratios in Experiment 3 is greater when delays are long than when they are short. However, only the decision model predicts the bitonic relationship effect of initial-link duration obtained by Christensen and Grace (2008).

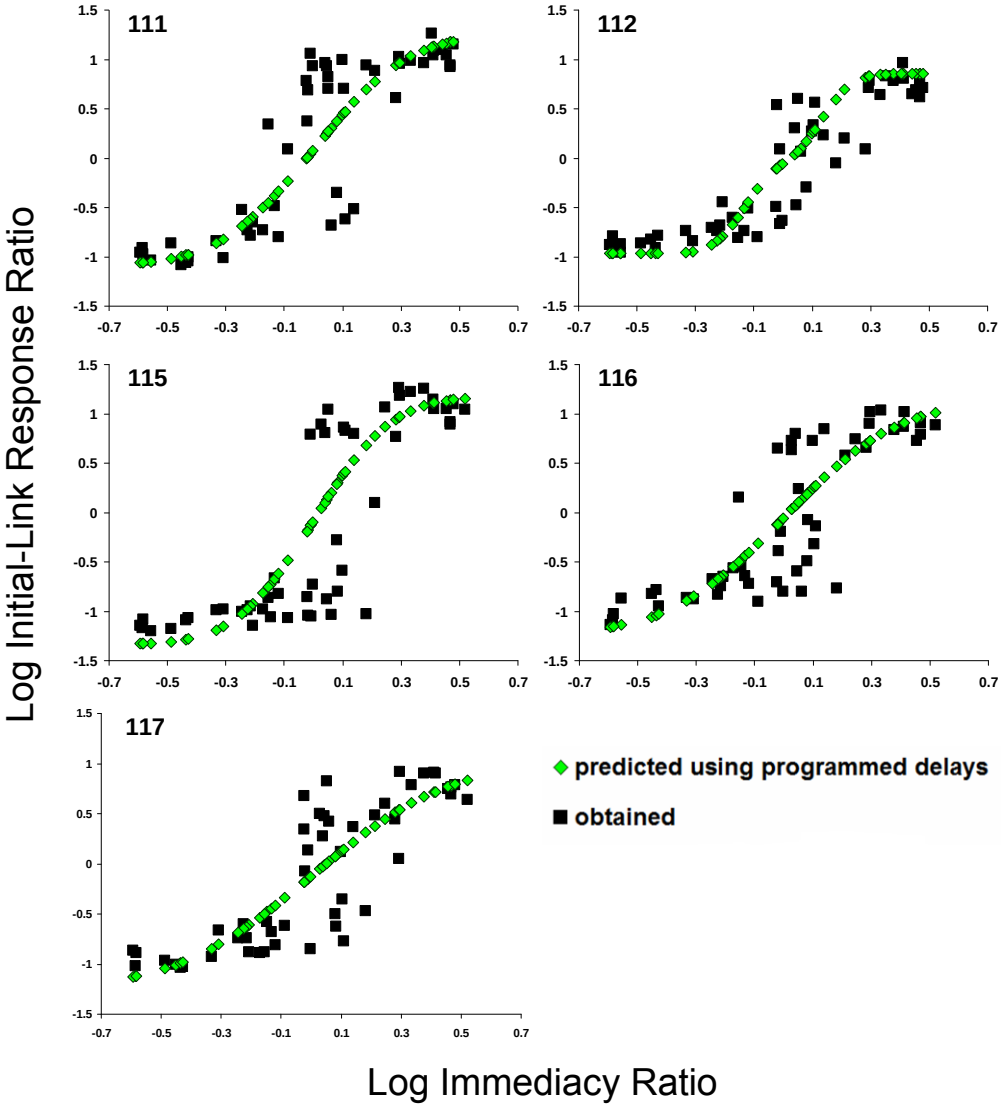

Figure 6.3. Log initial-link response ratios as a function of log immediacy ratios for the maximal-variation condition. Black squares and green diamonds represent obtained and predicted performance from the second half of single sessions, respectively.

Figure 6.3 shows obtained and predicted log response ratios from the maximal-variation condition of Experiment 1, calculated over the second half of sessions, as a function of log immediacy ratio. For all subjects, the decision model predicted that response allocation was a sigmoidal function of log immediacy ratio in the maximal-variation condition. These sigmoids are a qualitative improvement over a linear function (such as generalized matching). However, sigmoidal functions do not capture the pattern of response allocation fully: for all subjects, residual variability was a function of log

immediacy ratio: the decision model consistently underpredicted strength of response allocation when delays were similar.

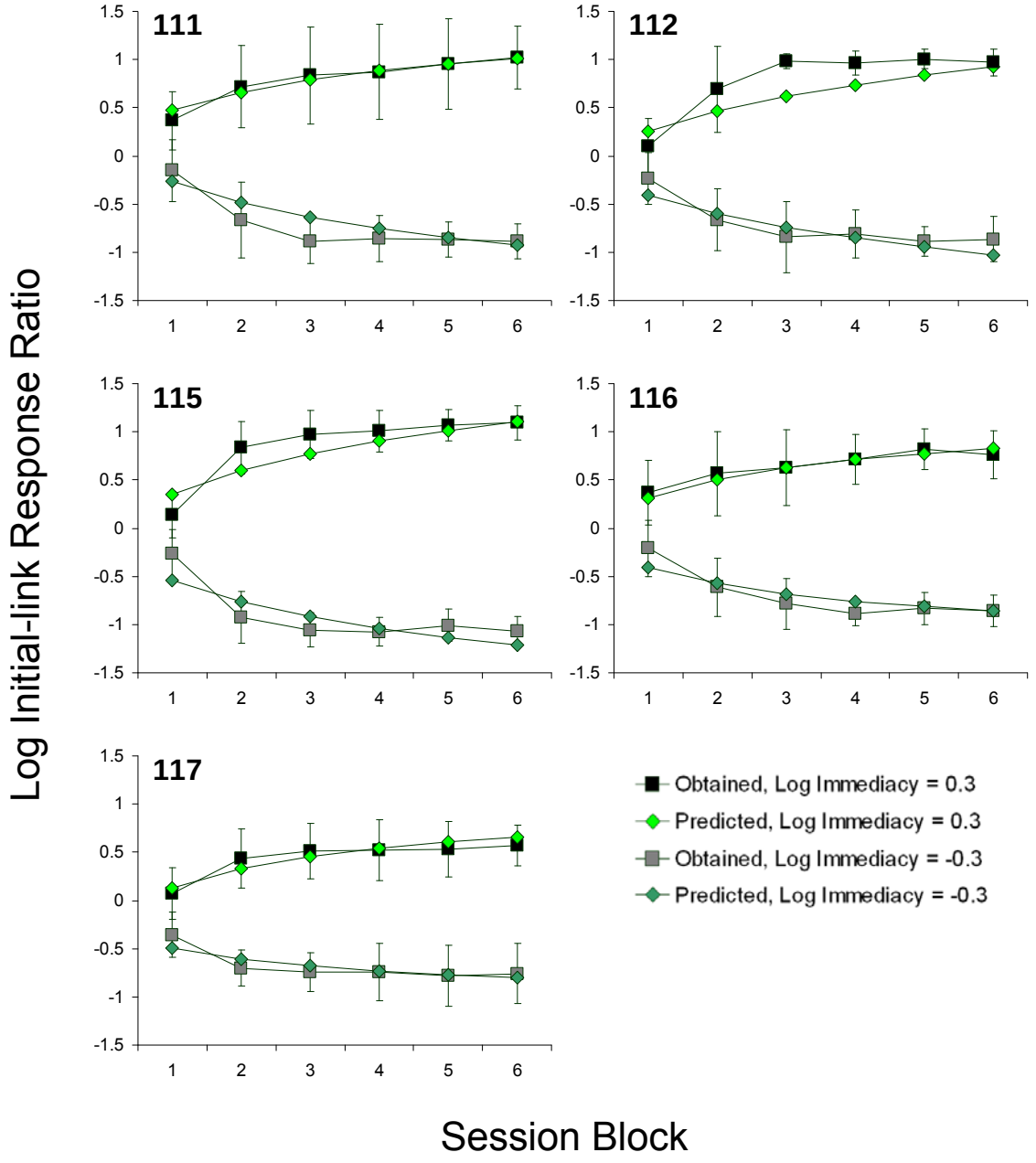

Figure 6.4. Log initial-link response ratios as a function of within-session block in the minimal-variation condition. Black and grey squares and green diamonds respectively represent obtained and predicted mean data (bars show obtained standard deviation) from each block. Data are plotted separately for sessions in which immediacy ratios were 1:2 and 2:1.

A critical advantage of the decision model is that it predicts expected trajectories of within-session acquisition of preference in rapid acquisition procedures. Figure 6.4 shows average obtained and predicted log initial-link response ratios from the minimal-variation plotted as a function of block. Obtained response allocation tended to be more extreme than predicted response allocation in blocks 2-5, indicating that responding stabilized more rapidly than predicted by the decision model. However, 58 out of 60 predicted values (2 immediacy ratios x 6 blocks x 5 subjects) were within a standard deviation of the mean obtained value. Similar results were obtained for Experiment 2 and for sensitivity coefficients from session blocks in Experiment 3 and the maximal-variation condition of Experiment 1. To a first approximation, the decision model provides a good account of the within-session changes in response allocation in this procedure.

All told, decision-model predictions of log initial-link response ratios can account for some of the features of rapid-acquisition concurrent-chains performance not described by generalized matching. It can describe the bitonic effect of initial-link duration on sensitivity to immediacy when initial-link durations change systematically and the negative linear effect on sensitivity when they change pseudorandomly. It can predict greater sensitivity to relative immediacy with longer absolute delays. Importantly, it predicts changes in expected log response ratios from one block of 12 cycles to the next.

6.4 Multiple reinforcer dimensions and the decision model

In Experiments 4, 5 and 6, multiple reinforcer dimensions changed randomly and independently across sessions. Aggregated across sessions, response allocation was determined by all reinforcer dimensions and adjusted to the specific contingencies arranged within individual sessions. The decision model was developed to describe choice between

reinforcers that differ in delay, but does not address effects of other reinforcer variables such as magnitude and probability. Is it possible to incorporate other dimensions into a version of the decision model? Many different approaches to incorporating effects of multiple dimensions could produce sensitivity to multiple dimensions at the molar level. One possibility is that subjects make separate judgments by comparing outcomes to separate criterion values for each dimension. However, there is no evidence from Experiments 4-6 that subjects made independent 'decisions' about individual dimensions. Instead, pigeons appeared to integrate information from multiple reinforcer dimensions within individual temporal epochs.

The goal of this section is to modify and extend the decision model so that it can account for the results of Experiments 4-6. The model described here assumes that subjects make single judgments about each reinforcer based on all dimensions. Specifically, subjects make a 'decision' about whether the outcome just experienced in a terminal link was relatively favorable or unfavorable, compared to a criterion. In the original decision model, the criterion only represented delay. In the model proposed here, the criterion is an average expectancy of reward, with expectancy (i.e., value) determined jointly by different reinforcer dimensions. Expected response strength increases and decreases according to p_{fav}, the probability the subject judged particular alternative to be favorable relative to a criterion:

$$RS_{n+1} = RS_n + p_{fav} * \left(Max_{RS} - RS_n\right) * \Delta - \left(1 - p_{fav}\right) * \left(RS_n - Min_{RS}\right) * \Delta ,$$

Equation 6.5

in which p_{fav} is determined by comparing a weighted average of the log values from each relevant reinforcer dimension to a single, normally-distributed criterion with mean logC and standard deviation σ:

$$p_{fav} = \Phi(\sum_{i=1}^{n} \frac{1}{n} * \alpha_{X_i} \log X_i, \log C, \sigma) .$$

Equation 6.6

As in Equation 6.3, Φ is a cumulative normal distribution with mean *logC* and standard deviation σ. Here, the first argument in the equation represents the estimated value of the just-experienced alternative. This version of the decision model assumes that a weighted average of log values from individual dimensions determines the valuation. The principal assumption of the concatenated matching law (Baum & Rachlin, 1969) is that log ratios from multiple reinforcer dimensions have additive and independent effects on response allocation. In Equation 6.6, multiple reinforcer dimensions have additive and independent effects on response strength. Thus, this decision model accounts for effects of multiple reinforcer dimensions by embedding the principal assumption of concatenated matching into determination of the value that is compared to the criterion.

To calculate p_{short}, Equation 6.3 uses (1- Φ), the inverse of the cumulative normal distribution. By contrast, Equation 6.6 uses Φ, the cumulative normal distribution. This is because although longer delays result in overall lower reinforcer access, larger values for most reinforcer dimensions (including rate, magnitude and probability) increase overall reinforcer access. (Neuringer (1967) used the words "reinforcer access" to describe duration access to food per unit of time spent responding to obtain food). To produce typical effects of delay using the cumulative normal distribution rather than its inverse, log immediacies rather than log terminal-link delays, were entered as terms in the model. This version of the decision model was fit to data from the last 50 sessions of Experiments 4, 5, and 6. The relative weighting parameter for magnitude, α_M, was constrained to be 1.0, because it was the only dimension common to all three experiments. Allowing α_M to vary

typically overparameterized the model. There was no increase in VAC and α and σ parameters took on unrealistically large values for all subjects in Experiments 5 and 6, and for all subjects except Pigeon 193 in Experiment 4. In that Experiment, Pigeon 193's sensitivity to relative magnitude was very low, so $\alpha_{I/D}$ and α_M were both fitted for those data. For each Experiment, the criterion mean equaled the average log reinforcer access ratio.

Table 6.5. Decision model parameter estimates and variances accounted for (VAC) based on performance blocks 4-6 of sessions from Experiments 4-6. *α*s are relative weighting parameters for rate (α_R), immediacy ($\alpha_{I/D}$) and probability (α_P). The relative weighting parameter for magnitude, α_M, was fixed at 1.0 for all subjects except Pigeon 193, Experiment 4 (fitted α_M = 0.10). Criterion mean (log*C*) was -0.29 in Experiment 4, -0.24 in Experiment 2 and 1.06 in Experiment 3.

Pigeon	α_R	$\alpha_{I/D}$	α_P	σ	Δ	Log*b*	VAC
			Experiment 4				
191	-	1.11	-	0.09	0.04	-0.02	0.52
192	-	1.81	-	0.15	0.53	-0.18	0.77
193	-	0.92	-	0.07	0.67	0.19	0.79
194	-	1.68	-	0.14	0.49	-0.13	0.78
			Experiment 5				
191	-	0.86	0.96	0.01	0.12	0.12	0.84
192	-	1.22	1.54	0.06	0.16	-0.14	0.71
193	-	1.26	1.23	0.07	0.28	-0.04	0.81
194	-	1.10	1.29	0.03	0.17	0.06	0.96
			Experiment 6				
195	1.09	-	-	0.08	0.06	0.01	0.86
196	1.19	-	-	0.18	0.04	0.06	0.73
197	1.10	-	-	0.10	0.06	-0.06	0.81
198	0.47	-	-	0.33	0.62	0.26	0.75

Table 6.5 shows parameter estimates and VAC for the decision model using programmed reinforcer values. Across sessions, the values that would determine the criterion were constant, so β was not used. Values for other parameters were comparable to those obtained in Experiments 1-3, and VAC was comparable to generalized-matching VAC reported in Tables 5.1 and 5.3. The absolute values of α parameters indicate the relative influence of each individual dimension. Values greater than 1 indicate that a dimension had greater influence on responding than magnitude. With two exceptions

(Pigeon 191, probability and Pigeon 194, delay in Experiment 2), α parameter estimates in were consistent with estimates of generalized-matching sensitivity; if the sensitivity coefficient for a particular dimension (as determined by multiple regression) was greater than the coefficient for Lag 0 log magnitude ratio, the α parameter for that dimension was greater than 1 in Table 6.5. Parameter estimates and VACs indicate that this version of the decision model can predict molar-level results of Experiments 4-6 successfully.

This decision model is able to account for different levels of preference obtained in the different configurations of log rate, immediacy, magnitude and probability ratios in Experiments 4-6. It also describes within-session changes in level of preference aggregated across sessions with the same configuration. Figure 6.5 shows obtained and predicted log initial-link response ratios for each block of 12 cycles from the last 50 sessions of Experiment 4 for individual subjects. Ninety five of 96 predictions (6 blocks x 4 configurations x 4 subjects) were within one standard deviation of the mean obtained log response ratio, and on average, predicted and obtained log ratios differed by 0.08 (*SE* = 0.02).

This expectancy decision model is the first quasidynamic model of choice to describe effects of multiple reinforcer dimensions on log initial-link response ratios. It accounts for the major features of stable performance in rapid acquisition concurrent chains and concurrent schedules and predicts expected change in log response ratios over the course of a session. Although it does not explain the abrupt nature of changes in response allocation within individual sessions, it nevertheless represents an important step towards understanding the dynamics of complex choice.

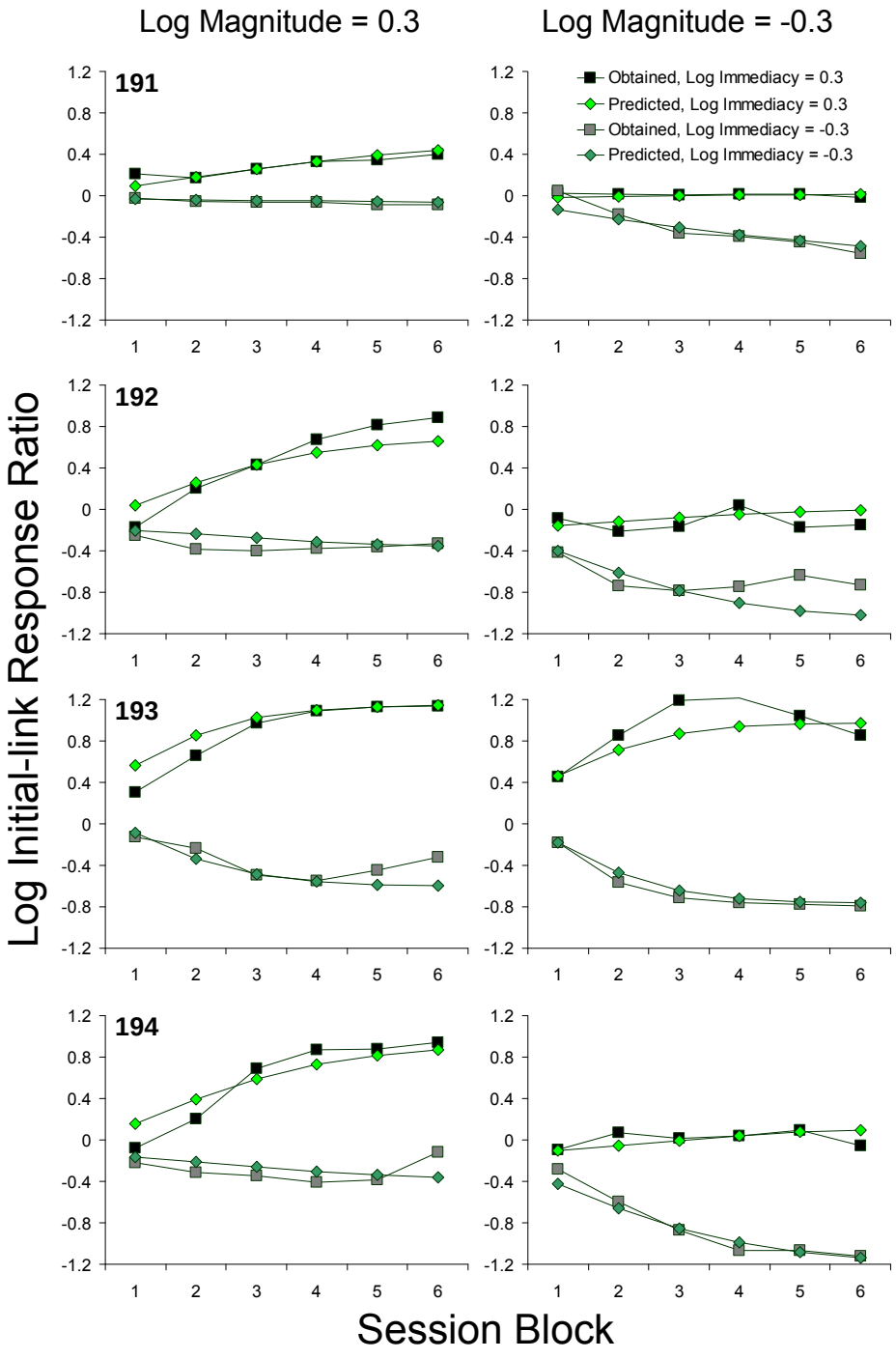

Figure 6.5. Log initial-link response ratios as a function of within-session block in Experiment 4. Black squares and green diamonds respectively represent obtained and prediction represent mean data from each block. Left and right panels show data for sessions in which magnitude ratios were 2:1 and 1:2 for individual subjects. Data are plotted separately for sessions in which immediacy ratios were 2:1 and 1:2.

6.5 Implications about the process(es) generating choice

Accounts of choice based on conditioned reinforcement and those derived from cognitive theories of timing make different assumptions about the nature of the relationship between response allocation and relative immediacy in concurrent chains. RET makes predictions about how behavior adapts to changes in terminal-link delay; specifically, that adaptation is abrupt and occurs at a stochastically determined point after a change in delays. The decision model proposed by Grace and colleagues (Grace & McLean, 2006; also Christensen & Grace, 2008, 2009a,b) and extended here describes a linear-operator process that produces gradual changes in relative expectancy. The major goal of this research was to determine whether matching to relative value or winner-take-all sampling – or some alternative process – describes the generation of concurrent-chains choice.

6.5.1 Choice as a function of log immediacy ratio

The discontinuous relationship between log initial-link response allocation and log immediacy ratio described by a piecewise-linear model (Equations 2.5 and 4.5) in Experiments 1 and 3 suggests that responding in maximal-variation rapid acquisition procedures was to some extent categorical. The source of the discontinuity was occasional sessions in which preference for the longer terminal link persisted throughout the session. Subjects usually preferred the shorter terminal-link delay in the second half of sessions. Preference for the longer terminal link was rare, but most likely to occur in sessions in which delays were similar. In a discrete-trial choice task in which reinforcer rate and magnitude were changed across blocks of trials, Lau and Glimcher (2005) found that monkeys perseverated in their choices; past reinforcement outcomes and past choices were both necessary to predict future choice behavior. Pigeons' occasional persistent preference

for the longer terminal-link delay in Experiments 1-3 is consistent with the monkeys' behavior. It is possible performance in these sessions reflected 'misclassifications' of relative terminal-link delays; subjects were making 'errors' in preference, then perseverating.

Whatever caused the observed discontinuity, it constitutes support for the RET assumption of an opting process and disconfirms matching to relative value. Theories of conditioned reinforcement predict a linear relationship between log initial-link response ratios and log immediacy ratios. The decision model can account for the observed nonlinearity in maximal-variation conditions of Experiments 1 and 3. For small σ parameters, the decision model predicts that log response ratios are a sigmoidal function of log immediacy ratios. If the relationship between response and immediacy ratios were truly sigmoidal rather than piecewise-linear, it could be argued that the result only disconfirmed that the "correct transformation" (Killeen, 1972) of relative immediacy is logarithmic. However, no transformation of relative immediacy will predict the clustering of response ratios observed in Experiments 1 and 3. For any transformation, residual error will always be systematic because the probability of misclassification is a function of relative immediacy. Hence, the discontinuity is positive evidence supporting the winner-take-all sampling hypothesis that underscores assumptions of theories of timing applied to choice.

6.5.2 Choice and timing

One admittedly mentalistic interpretation of the misclassifications made by subjects in these experiments is that subjects make errors in perceiving or remembering intervals between time markers. If imperfectly remembered delays (or delays sampled from memory) controlled pigeons' initial-link responding, misclassifications would be more

frequent for similar than for dissimilar pairs of delays (which was the case in Experiments 1 and 3). If choice performance and timing performance are both governed by representations of delay, it is worth asking if they are governed by the same representation. The positive residual covariation of log initial-link response ratios and log start and stop time ratios in Experiments 1-3 is evidence that, if a representation of delay governs choice and timing behaviors, then it is a common representation.

Is there a mechanistic or otherwise non-mentalistic interpretation of the obtained residual covariation? Quantitatively, it implies that learning error, ε_{REPi} in Equation 6.1, is nonzero and common to measures of choice and of timing. Perhaps a type of feed-forward mechanism (Gallistel, 2005) whereby a subject's own past behavior determines its future behavior (Lau & Glimcher, 2005) could account for the perseveration of 'irrational' preference for the longer delay, without appeal to internal representations. However, it would still require an explanation of how or why errors might occur in the first place. The empirical results of Experiments 1-3 answer the question of whether the processes that determine choice and timing are independent or interdependent. Why they are interdependent remains an open question. The only plausible hypothesis that has been advanced appeals to an internal representation of delays in memory.

6.5.3 Choice when alternatives differ on multiple reinforcer dimensions

Experiments 4-6 represent a different approach to investigating how choice adapts in unpredictable, dynamic environments. Ratios for two or three reinforcer dimensions were 2:1 or 1:2. They changed across sessions according to independent, random sequences. Although Gallistel and Gibbon (2000) described a means by which RET could account for effects of magnitude on choice between delayed rewards, as a model RET does

not explicitly address effects of parametric manipulations of multiple reinforcer dimensions. In principle, theories of conditioned reinforcement incorporate the concatenated-matching assumption that reinforcer dimensions have independent, additive effects on log response ratios. Grace (1994, 1995) made this prediction explicit for CCM. Consistent with the concatenated-matching assumption, pigeons were sensitive (to varying degrees) to Lag 0 ratios of all manipulated reinforcer dimensions, when responding was aggregated across sessions and within individual temporal epochs.

In an extension of the decision model, combinations of log values from all manipulated dimensions combine to produce a single 'expectancy,' and the expectancy of each outcome is compared to a criterion that is a normal distribution of expectancy values. This extension can account for pigeons' performance in Experiments 4-6. The concept of expectancy, developed in theories of timing (Gibbon, 1977), is similar to the concept of value referred to in theoretical discussions of free-operant choice (Baum & Rachlin, 1979). Moore (2008, p. 641) called concept of value "an intervening, mediating, organismic variable," and criticized it as "regrettably mentalistic." However, results of Experiments 4-6 and the successful application of the decision model to them provide empirical evidence that, in this case, an intervening variable has predictive value. One way to think about the decision model is to assume that a subject *makes a decision* about the expectancy of each outcome by comparing them to a distribution of previously experienced expectancies represented in memory. Such an assumption is intuitively appealing and parsimonious but certainly mentalistic. However, the decision model makes quantitative predictions about change in response strength as a function of a learning rate parameter, a value determined by a weighted combination of delay or immediacy, magnitude and other dimensions, and the probability that value is either greater than a criterion value. The criterion value is fixed

or it changes according to changes in reinforcer contingencies. Although the expectancy is an intervening variable, one need not interpret it as mentalistic.

6.5.4 Choice adaptation across temporal epochs

A conspicuous advantage of the decision model and RET applied to choice over accounts of choice based on conditioned reinforcement is that they can describe changes in response allocation over time. RET predicts that changes in behavior should be abrupt, and that a Poisson process determines the time of occurrence of abrupt changes. By contrast, in the decision model, a linear-operator process predicts gradual, monotonically increasing, negatively accelerated changes to expected response strength and response allocation. Although RET and the decision model describe fundamentally different processes, they are not mutually exclusive. In fact, aggregated across sessions, the stochastically occurring abrupt changes assumed by RET should produce a monotonically increasing, negatively accelerated function. Predictions of both models were correct for performance in Experiments 1-6; changes in response allocation tended to be abrupt (as in Figures 2.3 and 5.4) as predicted by RET. The described the acquisition trajectory of mean response allocation as a function of session block for negative and positive log immediacy ratios in minimal-variation conditions (and sensitivity to log immediacy ratio in maximal-variation conditions) very well.

Gallistel, Fairhurst and Balsam (2004) noted that abrupt changes to individual-subject learning curves are the norm for a variety of operant and classically conditioned behaviors. They list several implications of this observation. The ubiquity of abrupt changes in behavior conflicts with the assumption that ‘learning’ is the result of a gradual strengthening of associative connections. It implies that performance does not approach an

asymptote; instead, it fluctuates about an equilibrium. Gallistel et al. question the utility and validity of describing behavior change in terms of psychometric functions based on aggregated or group data. They suggested analyses of change-point latency (e.g., Tables 5.2 and 5.4) and comparisons of pre- and postacquisition rates of behavior as meaningful measures of learning. It is clear from cumulative response scatterplots from individual sessions that, although the decision model provides a solid quantitative description of average response allocation in each session block, the gradual changes produced by its linear-operator process do not occur in individual sessions.

6.5.5 Utility of the decision model as a quasidynamic theory of choice

The empirical results of these six experiments were broadly consistent with the assumptions of RET; log initial-link response ratios were a discontinuous, categorical function of log immediacy ratios, choice and timing were interdependent over and above their respective relationships with relative immediacy, and within-session changes in response allocation tended to be abrupt. However, RET does not generate closed-form predictions about response allocation and there is nothing that can be fitted to results to evaluate its effectiveness quantitatively. Although the linear-operator mechanism of the decision model seems to be at odds with observed abrupt changes in behavior, it is compatible with cognitive theories of timing applied to choice in many ways. In the decision model, the process that generates choice behavior involves a comparison between the delay (or expectancy) of a just-experienced outcome and a criterion. The decision model describes the behavioral phenomena explained by theories of conditioned reinforcement (initial- and terminal-link effects, independent effects of multiple dimensions). There is empirical support for one unique prediction of the decision model, a

bitonic effect of initial-link duration on sensitivity of log response ratios to log immediacy ratios (Christensen & Grace, 2008).

The linear-operator process of increasing or decreasing expected response strength for an alternative as a function of the probability delay or expectancy for the alternative exceeds that of a criterion does describe the aggregate within-session changes that occur in rapid acquisition concurrent chains procedures. An important failing of the decision model (in its current state) is that it does not predict the abrupt changes to response allocation that occur in individual sessions. However, the ratio comparator assumed by RET could be incorporated into the decision model. Within a terminal link, RET predicts that responding at a high rate starts when the ratio of current to baseline expectancy (which is positively correlated with immediacy and increases as a function of time elapsed since terminal-link onset) exceeds a threshold. In Experiments 1-6, terminal links change unpredictably across sessions. The decision model could append an assumption that within individual sessions, a ratio comparison of response allocation to current relative response strength is constantly recomputed. It could predict that abrupt changes in response allocation will occur when the ratio comparison falls outside of a threshold range ($\theta_{lower} - \theta_{upper}$):

$$\theta_{lower} \leq \log\frac{B_L}{B_R} \Big/ \log\frac{RS_L}{RS_R} \leq \theta_{upper}.$$

Equation 6.7

Gallistel at al (2004) pointed out that theoretical descriptions of average latency to change point and comparisons of pre- and posttransition response allocation may be more descriptive and theoretically valuable than models that predict the trajectory of average behavior based on performance aggregated across sessions. By incorporating a ratio-comparator into the decision model, it may be possible for the decision model to do both.

6.5.6 What to do with the time that is given us: possible avenues of future research

The word ‘integration’ is used in two ways with respect to choice and timing in this research. Integration can be something that the subject does (or does not do): pigeons integrate information from multiple reinforcer dimensions, or their choice and timing behaviour can be integrated (as opposed to independent). Integration can also be something that a theoretical or quantitative model does by providing theoretical principles or assumptions that can be applied to a variety of behavioral phenomena. An empirical goal of this thesis was to determine whether a common representation (or atheoretical single contruct) of delay was integrated in choice and timing behaviors. This question was answered affirmatively by the residual covariation of log initial-link response and start or stop time ratios.

The development of a quantitative, quasidynamic model for behavior that integrates choice and temporal control is a possible goal for the future. Gallistel and Gibbon (2000) outlined some important theoretical assumptions, generally supported empirically by results of Experiments 1-3, which describe relationships and interrelationships between choice, timing, and reinforcer contingencies. However, closed-form predictions of choice and temporal dynamics based on a single quantitative account of behavior would be an important advancement. An extended, modified version of the decision model could incorporate predictions about timing behavior in a manner consistent with assumptions of cognitive theories of timing. Further investigation of the abrupt nature of changes in initial- and terminal-link performance could distinguish possible mechanisms for integration.

References cited

Abarca, N., & Fantino, E. (1982). Choice and foraging. *Journal of the Experimental Analysis of Behavior*, 38, 117-123.

Adams, C. D., & Dickinson, A. (1981). Instrumental responding following reinforcer devaluation. *Quarterly Journal of Experimental Psychology B: Comparative and Physiological Psychology, 33B*, 109-121.

Alsop, B., & Davison, M. (1988). Concurrent-chain performance: Effects of absolute and relative terminal-link entry frequency. *Journal of the Experimental Analysis of Behavior, 49*, 351-365.

Alsop, B., & Elliffe, D. (1988). Concurrent-schedule performance: Effects of relative and overall reinforcer rate. *Journal of the Experimental Analysis of Behavior, 49*, 21-36.

Anderson, N. H. (1962). Application of an additive model to impression formation. *Science, 138*, 817-818.

Anderson, N. H. (1968). A simple model for information integration. In R. P. Abelson, E. Aronson, W. J. McGuire, T. M. Newcomb, M. J. Rosenberg & P. H. Tannenbaum (Eds.), *Theories of Cognitive Consistency: A Sourcebook* (pp. 731-743). Chicago: Rand McNally and Company.

Autor, S. M. (1960/1969). The strength of conditioned reinforcers as a function of frequency and probability of reinforcement. In D. P. Hendry (Ed.), *Conditioned Reinforcement* (pp. 127-162). Homewood IL: The Dorsey Press.

Bailey, J. T., & Mazur, J. E. (1990). Choice behavior in transition: Development of preference for the higher probability of reinforcement. *Journal of the Experimental Analysis of Behavior, 53*, 409-422.

Baum, W. M. (1974). On two types of deviation from the matching law: Bias and undermatching. *Journal of the Experimental Analysis of Behavior, 22*, 231-242.

Baum, W. M. (1979). Matching, undermatching, and overmatching in studies of choice. *Journal of the Experimental Analysis of Behavior, 32*, 269-281.

Baum, W. M. (2005). Understanding behaviorism: Behavior, culture, and evolution (2nd ed.). *Malden, MA, US: Blackwell Publishing, 312*.

Baum, W. M., & Davison, M. (2004). Choice in a variable environment: Visit patterns in the dynamics of choice. *Journal of the Experimental Analysis of Behavior, 81*, 85-127.

Baum, W. M., & Davison, M. (2009). Modeling the dynamics of choice. *Behavioural Processes, 81*, 189-194.

Baum, W. M., & Rachlin, H. C. (1969). Choice as time allocation. *Journal of the Experimental Analysis of Behavior, 12*, 861-874.

Baum, W. M., Schwendiman, J. W., & Bell, K. E. (1999). Choice, contingency discrimination, and foraging theory. *Journal of the Experimental Analysis of Behavior, 71*, 355-373.

Belke, T. W., Pierce, W. D., & Powell, R. A. (1989). Determinants of choice for pigeons and humans on concurrent-chains schedules of reinforcement. *Journal of the Experimental Analysis of Behavior, 52*, 97-109.

Berg, M. E., & Grace, R. C. (2004). Independence of terminal-link entry rate and immediacy in concurrent chains. *Journal of the Experimental Analysis of Behavior, 82*, 235-251.

Berg, M. E., & Grace, R. C. (2006). Initial-link duration and acquisition of preference in concurrent chains. *Learning & Behavior, 34*, 50-60.

Brownstein, A. J., & Pliskoff, S. S. (1968). Some effects of relative reinforcement rate and changeover delay in response-independent concurrent schedules of reinforcement. *Journal of the Experimental Analysis of Behavior, 11*, 683-688.

Brunner, D., Gallistel, C. R., Fairhurst, S., & Gibbon, J. (2000). Pretraining determines choice strategy in concurrent choice.

Bush, R. R., & Frederick, M. (1955). *Stochastic models for learning*. Oxford England: Wiley.

Bush, R. R., & Mosteller, F. (1951). A mathematical model for simple learning. *Psychological Review, 58*, 313-323.

Catania, A. C. (1963). Concurrent performances: A baseline for the study of reinforcement magnitude. *Journal of the Experimental Analysis of Behavior, 6*, 299-300.

Catania, A. C. (1970). Reinforcement schedules and psychophysical judgements: A study of some temporal properties of behavior In W. N. Schoenfeld (Ed.), *The theory of reinforcement schedules* (pp. 1-42). New York: Appleton-Century-Crofts.

Catania, A. C., & Reynolds, G. S. (1968). A quantitative analysis of the responding maintained by interval schedules of reinforcement. *Journal of the Experimental Analysis of Behavior, 11*, 327-383.

Cerutti, D. T., & Staddon, J. E. R. (2004). Time and rate measures in choice transitions.

Journal of the Experimental Analysis of Behavior, 81, 135-154.

Chelonis, J. J., & Logue, A. W. (1996). Effects of response type on pigeons' sensitivity to variation in reinforcer amount and reinforcer delay. *Journal of the Experimental Analysis of Behavior, 66*, 297-309.

Chelonis, J. J., & Logue, A. W. (1997). Effects of reinforcer type on rats' sensitivity to variation in reinforcer amount and reinforcer delay. *Behavioural Processes, 39*, 187-203.

Cheng, K., & Westwood, R. (1993). Analysis of single trials in pigeons' timing performance. *Journal of Experimental Psychology: Animal Behavior Processes, 19*, 56-67.

Cheng, K., Westwood, R., & Crystal, J. D. (1993). Memory variance in the peak procedure of timing in pigeons. *Journal of Experimental Psychology: Animal Behavior Processes, 19*, 68-76.

Christensen, D. R., & Grace, R. C. (2008). Rapid acquisition in concurrent chains: Effects of initial-link duration. *Behavioural Processes, 78*, 217-223.

Christensen, D. R., & Grace, R. C. (2009a). Response allocation in a rapid-acquisition concurrent-chains procedure: Effects of overall terminal-link duration. *Behavioural Processes, 81*, 233-237.

Christensen, D. R., & Grace, R. C. (2009b). Response allocation in concurrent chains when terminal-link delays follow an ascending and descending series. *Journal of the Experimental Analysis of Behavior, 91*, 1-20.

Chung, S.-H., & Herrnstein, R. J. (1967). Choice and delay of reinforcement. *Journal of the Experimental Analysis of Behavior, 10*, 67-74.

Church, R. M. (2002). Temporal learning. In H. Pashler & R. Gallistel (Eds.), *Steven's handbook of experimental psychology (3rd ed.), Vol. 3: Learning, motivation, and emotion.* (pp. 365-393): John Wiley & Sons, Inc, Hoboken, NJ, US.

Church, R. M., & Broadbent, H. A. (1990). Alternative representations of time, number, and rate. *Cognition. Special Issue: Animal cognition, 37*, 55-81.

Church, R. M., Meck, W. H., & Gibbon, J. (1994). Application of scalar timing theory to individual trials. *Journal of Experimental Psychology: Animal Behavior Processes, 20*, 135-155.

Crowley, M. A., & Donahoe, J. W. (2004). Matching: Its acquisition and generalization. *Journal of the Experimental Analysis of Behavior, 82*, 143-159.

Crystal, J. D., Church, R. M., & Broadbent, H. A. (1997). Systematic nonlinearities in the memory representation of time. *Journal of Experimental Psychology: Animal Behavior Processes, 23*, 267-282.

Davison, M. (1983). Bias and sensitivity to reinforcement in a concurrent-chain schedule. *Journal of the Experimental Analysis of Behavior, 40*, 15-34.

Davison, M. (1988). Concurrent schedules: Interaction of reinforcer frequency and reinforcer duration. *Journal of the Experimental Analysis of Behavior, 49*, 339-349.

Davison, M. (1991). Choice, changeover, and travel: A quantitative model. *Journal of the Experimental Analysis of Behavior, 55*, 47-61.

Davison, M., & Baum, W. M. (2000). Choice in a variable environment: Every reinforcer counts. *Journal of the Experimental Analysis of Behavior, 74*, 1-24.

Davison, M., & Baum, W. M. (2002). Choice in a variable environment: Effects of blackout duration and extinction between components. *Journal of the Experimental Analysis of Behavior, 77*, 65-89.

Davison, M., & Baum, W. M. (2003). Every reinforcer counts: Reinforcer magnitude and local preference. *Journal of the Experimental Analysis of Behavior, 80*, 95-119.

Davison, M., & Baum, W. M. (2006). Do conditional reinforcers count? *Journal of the Experimental Analysis of Behavior, 86*, 269-283.

Davison, M., & Baum, W. M. (2007). Local effects of delayed food. *Journal of the Experimental Analysis of Behavior, 87*, 241-260.

Davison, M., & Jenkins, P. E. (1985). Stimulus discriminability, contingency discriminability, and schedule preformance. *Animal Learning & Behavior, 13*, 77-84.

Davison, M., & Jones, B. M. (1995). A quantitative analysis of extreme choice. *Journal of the Experimental Analysis of Behavior, 64*, 147-162.

Davison, M., & McCarthy, D. (1988). The matching law: A research review. *Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc, 285*.

Davison, M. C. (1976). Preference for fixed-interval schedules: Effects of unequal initial links. *Journal of the Experimental Analysis of Behavior, 25*, 371-376.

Davison, M. C. (1987). The analysis of concurrent-chain performance. In M. L. Commons, J. E. Mazur, J. A. Nevin & H. Rachlin (Eds.), *The effect of delay and of intervening events on reinforcement value. Quantitative analyses of behavior, Vol. 5.* (pp. 225-241). Hillsdale, NJ, England: Lawrence Erlbaum Associates, Inc.

Davison, M. C., & Temple, W. (1973). Preference for fixed-interval schedules: An alternative model. *Journal of the Experimental Analysis of Behavior. Vol., 20*, 393-403.

Dews, P. B. (1970). The theory of fixed-interval responding. In W. N. Schoenfeld (Ed.), *The theory of reinforcement schedules* (pp. 43-62). New York: Appleton-Century-Crofts.

Donahoe, J. W., & Palmer, D. C. (1994). *Learning and complex behavior*. Needham Heights, MA US: Allyn & Bacon.

Elliffe, D., & Alsop, B. (1996). Concurrent choice: Effects of overall reinforcer rate and the temporal distribution of reinforcers. *Journal of the Experimental Analysis of Behavior, 65*, 445-463.

Elliffe, D., Davison, M., & Landon, J. (2008). Relative reinforcer rates and magnitudes do not control concurrent choice independently. *Journal of the Experimental Analysis of Behavior, 90*, 169-185.

Estes, W. K. (1950). Toward a statistical theory of learning. *Psychological Review, 57*, 94-107.

Fantino, E. (1969). Choice and rate of reinforcement. *Journal of the Experimental Analysis of Behavior, 12*, 723-730.

Fantino, E., & Abarca, N. (1985). Choice, optimal foraging, and the delay-reduction hypothesis. *Behavioral and Brain Sciences*, *8*, 315-362.

Fantino, E., & Royalty, P. (1987). A molecular analysis of choice on concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior, 48*, 145-159.

Fantino, E., Squires, N., Delbruck, N., & Peterson, C. (1972). Choice behavior and the accessibility of the reinforcer. *Journal of the Experimental Analysis of Behavior, 18*, 35-43.

Ferster, C. B., & Skinner, B. F. (1957). *Schedules of reinforcement*. East Norwalk, CT US: Appleton-Century-Crofts.

Fleshler, M., & Hoffman, H. S. (1962). A progression for generating variable-interval schedules. *Journal of the Experimental Analysis of Behavior, 5*, 529-530.

Gallistel, C. R. (2005). Deconstructing the law of effect. *Games and Economic Behavior. Special Issue on Neuroeconomics, 52*, 410-423.

Gallistel, C. R., Fairhurst, S., & Balsam, P. (2004). The learning curve: Implications of a quantitative analysis. *Proceedings of the National Academy of Sciences of the*

United States of America, 101, 13124-13131.

Gallistel, C. R., & Gelman, R. (2000). Non-verbal numerical cognition: From reals to integers. *Trends in Cognitive Sciences, 4*(2), 59-65.

Gallistel, C. R., & Gibbon, J. (2000). Time, rate, and conditioning. *Psychological Review, 107*(2), 289-344.

Gallistel, C. R., & Gibbon, J. (2001). Computational versus associative models of simple conditioning. *Current Directions in Psychological Science, 10*(4), 146-150.

Gallistel, C. R., Mark, T. A., King, A. P., & Latham, P. E. (2001). The rat approximates an ideal detector of changes in rates of reward: Implications for the law of effect. *Journal of Experimental Psychology: Animal Behavior Processes, 27*, 354-372.

Garner, W. R., Hake, H. W., & Eriksen, C. W. (1956). Operationism and the concept of perception. *Psychological Review, 63*, 149-159.

Gibbon, J. (1971). Scalar timing and semi-Markov chains in free-operant avoidance. *Journal of Mathematical Psychology, 8*, 109-138.

Gibbon, J. (1972). Timing and discrimination of shock density in avoidance. *Psychological Review, 79*, 68-92.

Gibbon, J. (1977). Scalar expectancy theory and Weber's law in animal timing. *Psychological Review, 84*, 279-325.

Gibbon, J. (1991). Origins of scalar timing. *Learning and Motivation. Special Issue: Animal timing, 22*, 3-38.

Gibbon, J., & Church, R. M. (1990). Representation of time. *Cognition. Special Issue: Animal cognition, 37*, 23-54.

Gibbon, J., & Church, R. M. (1992). Comparison of variance and covariance patterns in parallel and serial theories of timing. *Journal of the Experimental Analysis of Behavior. Special Issue: Behavior dynamics, 57*, 393-406.

Gibbon, J., Church, R. M., Fairhurst, S., & Kacelnik, A. (1988). Scalar expectancy theory and choice between delayed rewards. *Psychological Review, 95*, 102-114.

Gigerenzer, G., & Goldstein, D. G. (1996). Reasoning the fast and frugal way: Models of bounded rationality. *Psychological Review, 103*, 650-669.

Grace, R. C. (1994). A contextual model of concurrent-chains choice. *Journal of the Experimental Analysis of Behavior, 61*, 113-129.

Grace, R. C. (1995). Independence of reinforcement delay and magnitude in concurrent chains. *Journal of the Experimental Analysis of Behavior, 63*, 255-276.

Grace, R. C. (1999). The matching law and amount-dependent exponential discounting as accounts of self-control choice. *Journal of the Experimental Analysis of Behavior, 71*, 27-44.

Grace, R. C. (2001). On the failure of operationism. *Theory & Psychology, 11*, 5-33.

Grace, R. C. (2002a). Acquisition of preference in concurrent chains: Comparing linear-operator and memory-representational models. *Journal of Experimental Psychology: Animal Behavior Processes, 28*, 257-276.

Grace, R. C. (2002b). The value hypothesis and acquisition of preference in concurrent chains. *Animal Learning & Behavior, 30*, 21-33.

Grace, R. C., Bedell, M. A., & Nevin, J. A. (2002). Preference and resistance to change with constant- and variable-duration terminal links: Independence of reinforcement rate and magnitude. *Journal of the Experimental Analysis of Behavior, 77*, 233-255.

Grace, R. C., Berg, M. E., & Kyonka, E. G. E. (2006). Choice and timing in concurrent chains: Effects of initial-link duration. *Behavioural Processes, 71*, 188-200.

Grace, R. C., & Bragason, O. (2005). Does sensitivity to magnitude depend on the temporal distribution of reinforcement?: Erratum. *Journal of the Experimental Analysis of Behavior, 83*, 220.

Grace, R. C., Bragason, O., & McLean, A. P. (2003). Rapid Acquisition of Preference in Concurrent Chains. *Journal of the Experimental Analysis of Behavior, 80*, 235-252.

Grace, R. C., & McLean, A. P. (2006). Rapid acquisition in concurrent chains: Evidence for a decision model. *Journal of the Experimental Analysis of Behavior, 85*, 181-202.

Grace, R. C., & Nevin, J. A. (1999). Timing and choice in concurrent chains. *Behavioural Processes. Special Issue: Interval timing: Is there a clock? Part II, 45*, 115-127.

Grace, R. C., & Savastano, H. I. (2000). Temporal context and conditioned reinforcement value. *Journal of Experimental Psychology: General, 129*, 427-443.

Green, L., & Estle, S. J. (2003). Preference reversals with food and water reinforcers in rats. *Journal of the Experimental Analysis of Behavior, 79*, 233-242.

Guilhardi, P., & Church, R. M. (2005). Dynamics of temporal discrimination. *Learning & Behavior, 33*, 399-416.

Hamblin, R. L., & Miller, H. L. (1977). Matching as a multivariate power law: Frequency of behavior versus frequency and magnitude of reinforcement. *Learning and Motivation, 8*, 113-125.

Herrnstein, R. J. (1961). Relative and absolute strength of response as a function of

frequency of reinforcement. *Journal of the Experimental Analysis of Behavior, 4*, 267-272.

Herrnstein, R. J. (1964). Secondary reinforcement and rate of primary reinforcement. *Journal of the Experimental Analysis of Behavior, 7*, 27-36.

Herrnstein, R. J. (1970). On the law of effect. *Journal of the Experimental Analysis of Behavior, 13*, 243-266.

Herrnstein, R. J. (1982). Melioration as behavioral dynamism. In M. L. Commons, R. J. Herrnstein & H. Rachlin (Eds.), *Matching and Maximizing Accounts* (Vol. II, pp. 433-458). Cambridge, MA: Ballinger Publishing Company.

Higa, J. J. (1997). Rapid timing of a single transition in interfood interval duration by rats. *Animal Learning & Behavior, 25*, 177-184.

Higa, J. J., Thaw, J. M., & Staddon, J. E. (1993). Pigeons' wait-time responses to transitions in interfood-interval duration: Another look at cyclic schedule performance. *Journal of the Experimental Analysis of Behavior, 59*, 529-541.

Higa, J. J., Wynne, C. D., & Staddon, J. E. (1991). Dynamics of time discrimination. *Journal of Experimental Psychology: Animal Behavior Processes, 17*, 281-291.

Hollard, V., & Davison, M. C. (1971). Preference for qualitatively different reinforcers. *Journal of the Experimental Analysis of Behavior, 16*, 375-380.

Hunter, I., & Davison, M. (1982). Independence of response force and reinforcement rate on concurrent variable-interval schedule performance. *Journal of the Experimental Analysis of Behavior, 37*, 183-197.

Hunter, I., & Davison, M. (1985). Determination of a behavioral transfer function: White-noise analysis of session-to-session response-ratio dynamics on concurrent VI VI schedules. *Journal of the Experimental Analysis of Behavior, 43*, 43-59.

Innis, N. K., Mitchell, S. K., & Staddon, J. E. (1993). Temporal control on interval schedules: What determines the postreinforcement pause? *Journal of the Experimental Analysis of Behavior, 60*, 293-311.

Jozefowiez, J., Cerutti, D. T., & Staddon, J. E. R. (2005). Timing in Choice Experiments. *Journal of Experimental Psychology: Animal Behavior Processes, 31*, 213-225.

Jozefowiez, J., Cerutti, D. T., & Staddon, J. E. R. (2006). Timescale invariance and Weber's law in choice. *Journal of Experimental Psychology: Animal Behavior Processes, 32*, 229-238.

Killeen, P. (1968). On the Measurement of Reinforcement Frequency in the Study of

Preference. *Journal of the Experimental Analysis of Behavior, 11*, 263-269.

Killeen, P. (1972). The matching law. *Journal of the Experimental Analysis of Behavior, 17*, 489-495.

Killeen, P. R. (1982). Incentive theory: II. Models for choice. *Journal of the Experimental Analysis of Behavior, 38*, 217-232.

Killeen, P. R., Cate, H., & Tran, T. (1993). Scaling pigeons' choice of feeds: Bigger is better. *Journal of the Experimental Analysis of Behavior. Special Issue: The nature of reinforcement, 60*, 203-217.

Killeen, P. R., & Fetterman, J. G. (1988). A behavioral theory of timing. *Psychological Review, 95*, 274-295.

Kyonka, E. G. E. (2008). The matching law and effects of reinforcer rate and magnitude on choice in transition. *Behavioural Processes, 78*, 210-216. ***This publication appears as Experiment 6 in Chapter 5.***

Kyonka, E. G. E., & Grace, R. C. (2007). Rapid acquisition of choice and timing in pigeons. *Journal of Experimental Psychology: Animal Behavior Processes, 33*, 392-408. ***This publication appears as Experiment 1 in Chapter 2.***

Kyonka, E. G. E., & Grace, R. C. (2008). Rapid acquisition of preference in concurrent chains when alternatives differ on multiple dimensions of reinforcement. *Journal of the Experimental Analysis of Behavior, 89*, 49-69. ***This publication appears as Experiments 4 and 5 in Chapter 5.***

Kyonka, E. G. E., & Grace, R. C. (2009). Effects of unpredictable changes in initial-link duration on choice and timing. *Behavioural Processes, 81*, 227-232. ***This publication appears as Experiment 2 in Chapter 3.***

Landon, J., Davidson, M., & Elliffe, D. (2003). Concurrent schedules: Reinforcer magnitude effects. *Journal of the Experimental Analysis of Behavior, 79*, 351-365.

Lau, B., & Glimcher, P. W. (2005). Dynamic response-by-response models of matching behavior in rhesus monkeys. *Journal of the Experimental Analysis of Behavior, 84*, 555-579.

Leak, T. M., & Gibbon, J. (1995). Simultaneous timing of multiple intervals: Implications of the scalar property. *Journal of Experimental Psychology: Animal Behavior Processes, 21*, 3-19.

Lee, M. D., & Cummins, T. D. R. (2004). Evidence accumulation in decision making: Unifying the "take the best" and the "rational" models. *Psychonomic Bulletin &*

Review, 11, 343-352.

Lejeune, H., Ferrara, A., Simons, F. d. r., & Wearden, J. H. (1997). Adjusting to changes in the time of reinforcement: Peak interval transitions in rats. *Journal of Experimental Psychology: Animal Behavior Processes, 23*, 211-231.

Leon, M. I., & Gallistel, C. R. (1998). Self-stimulating rats combine subjective reward magnitude and subjective reward rate multiplicatively. *Journal of Experimental Psychology: Animal Behavior Processes, 24*, 265-277.

Logue, A. W. (1998). Self-control. In W. T. O'Donohue (Ed.), *Learning and behavior therapy*. (pp. 252-273). Needham Heights, MA, US: Allyn & Bacon.

Logue, A. W., & Chavarro, A. (1987). Effect on choice of absolute and relative values of reinforcer delay, amount, and frequency. *Journal of Experimental Psychology: Animal Behavior Processes, 13*, 280-291.

Logue, A. W., Forzano, L. B., & Tobin, H. (1992). Independence of reinforcer amount and delay: The generalized matching law and self-control in humans. *Learning and Motivation, 23*, 326-342.

Logue, A. W., Rodriguez, M. L., Peña-Correal, T. E., & Mauro, B. C. (1984). Choice in a self-control paradigm: Quantification of experience-based differences. *Journal of the Experimental Analysis of Behavior, 41*, 53-67.

MacEwen, D. (1972). The effects of terminal-link fixed-interval and variable-interval schedules on responding under concurrent chained schedules. *Journal of the Experimental Analysis of Behavior, 18*, 253-261.

Maguire, D. R., Hughes, C. E., & Pitts, R. C. (2007). Rapid acquisition of preference in concurrent schedules: Effects of reinforcement amount. *Behavioural Processes, 75*, 213-219.

Massaro, D. W., & Friedman, D. (1990). Models of integration given multiple sources of information. *Psychological Review, 97*, 225-252.

Mattson, K. M. (2005). *The value of signals for reward : choice in concurrent chains : a thesis submitted in partial fulfilment of the requirements for the degree of the Doctor of Philosophy in the University of Canterbury*. University of Canterbury, Christchurch.

Mazur, J. E. (1984). Tests of an equivalence rule for fixed and variable reinforcer delays. *Journal of Experimental Psychology: Animal Behavior Processes, 10*, 426-436.

Mazur, J. E. (1986). Fixed and variable ratios and delays: Further tests of an equivalence

rule. *Journal of Experimental Psychology: Animal Behavior Processes, 12*, 116-124.

Mazur, J. E. (1988). Choice between small certain and large uncertain reinforcers. *Animal Learning & Behavior, 16*, 199-205.

Mazur, J. E. (1991). Choice with probabilistic reinforcement: Effects of delay and conditioned reinforcers. *Journal of the Experimental Analysis of Behavior, 55*, 63-77.

Mazur, J. E. (1995). Development of preference and spontaneous recovery in choice behavior with concurrent variable-interval schedules. *Animal Learning & Behavior, 23*, 93-103.

Mazur, J. E. (1996). Past experience, recency, and spontaneous recovery in choice behavior. *Animal Learning & Behavior, 24*, 1-10.

Mazur, J. E. (2001). Hyperbolic value addition and general models of animal choice. *Psychological Review, 108*, 96-112.

Mazur, J. E. (2002). Evidence against a constant-difference effect in concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior, 77*, 147-155.

Mazur, J. E. (2004). Varying initial-link and terminal-link durations in concurrent-chains schedules: A comparison of three models. *Behavioural Processes, 66*, 189-200.

Mazur, J. E. (2005). Exploring a Concurrent-Chains Paradox: Decreasing Preference as an Initial Link Is Shortened. *Journal of Experimental Psychology: Animal Behavior Processes, 31*, 3-17.

Mazur, J. E., Blake, N., & McManus, C. (2001). Transitional choice behavior on concurrent-chain schedules. *Behavioural Processes, 53*, 171-180.

Mazur, J. E., Commons, M. L., Nevin, J. A., & Rachlin, H. (1987). An adjusting procedure for studying delayed reinforcement *The effect of delay and of intervening events on reinforcement value.* (pp. 55-73). Hillsdale, NJ England: Lawrence Erlbaum Associates, Inc.

Mazur, J. E., & Ratti, T. A. (1991). Choice behavior in transition: Development of preference in a free-operant procedure. *Animal Learning & Behavior, 19*, 241-248.

Mazur, J. E., & Romano, A. (1992). Choice with delayed and probabilistic reinforcers: Effects of variability, time between trials, and conditioned reinforcers. *Journal of the Experimental Analysis of Behavior, 58*, 513-525.

Mazur, J. E., Stellar, J. R., & Waraczynski, M. (1987). Self-control choice with electrical

stimulation of the brain as a reinforcer. *Behavioural Processes, 15*, 143-153.

McDevitt, M. A., & Williams, B. A. (2003). Arousal, changeover responses, and preference in concurrent schedules. *Journal of the Experimental Analysis of Behavior, 80*, 261-272.

McLean, A. P., & Blampied, N. M. (2001). Sensitivity to relative reinforcer rate in concurrent schedules: Independence from relative and absolute reinforcer duration. *Journal of the Experimental Analysis of Behavior, 75*, 25-42.

Meck, W. H., Komeily-Zadeh, F. N., & Church, R. M. (1984). Two-step acquisition: Modification of an internal clock's criterion. *Journal of Experimental Psychology: Animal Behavior Processes, 10*, 297-306.

MED-PC for Windows (Version 1.10). St. Albans, VT: Med Associates Inc.

Miller, H. L. (1976). Matching-based hedonic scaling in the pigeon. *Journal of the Experimental Analysis of Behavior, 26*, 335-347.

Moore, J. (2008). A critical appraisal of contemporary approaches in the quantitative analysis of behavior. *The Psychological Record, 58*, 641-664.

Myerson, J., & Green, L. (1995). Discounting of delayed rewards: Models of individual choice. *Journal of the Experimental Analysis of Behavior. Special Issue: Behavioral economics, 64*, 263-276.

Myerson, J., & Hale, S. (1988). Choice in transition: A comparison of melioration and the kinetic model. *Journal of the Experimental Analysis of Behavior, 49*, 291-302.

Myerson, J., & Miezin, F. M. (1980). The kinetics of choice: An operant systems analysis. *Psychological Review, 87*, 160-174.

Navarick, D. J., & Fantino, E. (1976). Self-control and general models of choice. *Journal of Experimental Psychology: Animal Behavior Processes, 2*, 75-87.

Neuringer, A. J. (1967). Effects of reinforcement magnitude on choice and rate of responding. *Journal of the Experimental Analysis of Behavior, 10*, 417-424.

Omino, T., & Ito, M. (1993). Choice and delay of reinforcement: Effects of terminal-link stimulus and response conditions. *Journal of the Experimental Analysis of Behavior, 59*, 361-371.

Ong, E. L., & White, K. G. (2004). Amount-dependent temporal discounting? *Behavioural Processes, 66*, 201-212.

Preston, R. A. (1994). Choice in the time-left procedure and in concurrent chains with a time-left terminal link. *Journal of the Experimental Analysis of Behavior, 61*, 349-

373.

Rachlin, H. (1971). On the tautology of the matching law. *Journal of the Experimental Analysis of Behavior, 15*, 249-251.

Rachlin, H., & Green, L. (1972). Commitment, choice and self-control. *Journal of the Experimental Analysis of Behavior, 17*, 15-22.

Rescorla, R. A., & Wagner, A. R. (1972). A theory of Pavlovian conditioning: Variations in the effectiveness of reinforcement and nonreinforcement. In A. H. Black & W. F. Prokasy (Eds.), *Classical Conditioning II: Current Research and Theory* (pp. 64-99). New York: Appleton-Century-Crofts.

Riccio, D. C., Rabinowitz, V. C., & Axelrod, S. (1994). Memory: When less is more. *American Psychologist, 49*, 917-926.

Roberts, S. (1981). Isolation of an internal clock. *Journal of Experimental Psychology: Animal Behavior Processes, 7*, 242-268.

Rodriguez, M. L., & Logue, A. W. (1986). Independence of the amount and delay ratios in the generalized matching law. *Animal Learning & Behavior, 14*, 29-37.

Rodriguez, M. L., & Logue, A. W. (1988). Adjusting delay to reinforcement: Comparing choice in pigeons and humans. *Journal of Experimental Psychology: Animal Behavior Processes, 14*, 105-117.

Schneider, B. A. (1969). A two-state analysis of fixed-interval responding in the pigeon. *Journal of the Experimental Analysis of Behavior, 12*, 677-687.

Schneider, J. W. (1968). Effects of percentage reinforcement on choice in a concurrent chain schedule. *Psychonomic Science, 12*, 211-212.

Schneider, J. W. (1973). Reinforcer effectiveness as a function of reinforcer rate and magnitude: A comparison of concurrent performances. *Journal of the Experimental Analysis of Behavior, 20*, 461-471.

Schofield, G., & Davison, M. (1997). Nonstable concurrent choice in pigeons. *Journal of the Experimental Analysis of Behavior, 68*, 219-232.

Silberberg, A., Hamilton, B., Ziriax, J. M., & Casey, J. (1978). The structure of choice. *Journal of Experimental Psychology: Animal Behavior Processes, 4*, 368-398.

Skinner, B. F. (1938). *The behavior of organisms: an experimental analysis*. Oxford England: Appleton-Century.

Skinner, B. F. (1950). Are theories of learning necessary? *Psychological Review, 57*, 193-216.

Skinner, B. F. (1971). *Beyond freedom and dignity*. New York, NY US: Knopf/Random House.

Spetch, M. L., & Dunn, R. (1987). Choice between reliable and unreliable outcomes: Mixed percentage-reinforcement in concurrent chains. *Journal of the Experimental Analysis of Behavior, 47*, 57-72.

Squires, N., & Fantino, E. (1971). A model for choice in simple concurrent and concurrent-chains schedules. *Journal of the Experimental Analysis of Behavior, 15*, 27-38.

Staddon, J. E. (1967). Attention and Temporal Discrimination: Factors Controlling Responding under a Cyclic-Interval Schedule. *Journal of the Experimental Analysis of Behavior, 10*, 349-359.

Staddon, J. E. (1988). Quasi-dynamic choice models: Melioration and ratio invariance. *Journal of the Experimental Analysis of Behavior, 49*, 303-320.

Staddon, J. E., Wynne, C. D., & Higa, J. J. (1991). The role of timing in reinforcement schedule performance. *Learning and Motivation. Special Issue: Animal timing, 22*, 200-225.

Staddon, J. E. R., & Cerutti, D. T. (2003). Operant conditioning. *Annual Review of Psychology, 54*, 115-144.

Staddon, J. E. R., & Higa, J. J. (1999). Time and memory: Towards a pacemaker-free theory of interval timing. *Journal of the Experimental Analysis of Behavior, 71*, 215-251.

Stubbs, D. A., & Pliskoff, S. S. (1969). Concurrent responding with fixed relative rate of reinforcement. *Journal of the Experimental Analysis of Behavior, 12*, 887-895.

Ta, W.-M., Pitts, R. C., Hughes, C. E., McLean, A. P., & Grace, R. C. (2008). Rapid acquisition of preference in concurrent chains: Effects of d-amphetamine on sensitivity to reinforcement delay. *Journal of the Experimental Analysis of Behavior, 89*, 71-91.

Taylor, R., & Davison, M. (1983). Sensitivity to reinforcement in concurrent arithmetic and exponential schedules. *Journal of the Experimental Analysis of Behavior, 39*, 191-198.

Todorov, J. C. (1973). Interaction of frequency and magnitude of reinforcement on concurrent performances. *Journal of the Experimental Analysis of Behavior, 19*, 451-458.

Vaughan, W. (1981). Melioration, matching, and maximization. *Journal of the*

Experimental Analysis of Behavior, 36, 141-149.

Vaughan, W. (1982). Choice and the Rescorla-Wagner model. In M. L. Commons, R. J. Herrnstein & H. Rachlin (Eds.), *Quantitative Analyses of Behavior: Matching and Maximizing Accounts* (Vol. 2, pp. 263-280). Cambridge, MA: Ballinger Publishing Company.

Vaughan, W. (1985). Choice: A local analysis. *Journal of the Experimental Analysis of Behavior, 43*, 383-405.

Williams, B. A. (1994). Reinforcement and choice. In N. J. Mackintosh (Ed.), *Animal learning and cognition. Handbook of perception and cognition series (2nd ed.) 3.* (pp. 81-108): Academic Press, Inc, San Diego, CA, US.

Williams, B. A., & Fantino, E. (1978). Effects on choice of reinforcement delay and conditioned reinforcement. *Journal of the Experimental Analysis of Behavior, 29*, 77-86.

Wynne, C. D., & Staddon, J. E. (1988). Typical delay determines waiting time on periodic-food schedules: Static and dynamic tests. *Journal of the Experimental Analysis of Behavior, 50*, 197-210.

Young, M. E., & Wasserman, E. A. (2002). Limited attention and cue order consistency affect predictive learning: A test of similarity measures. *Journal of Experimental Psychology: Learning, Memory, and Cognition, 28*, 484-496.

Zeiler, M. D., & Powell, D. G. (1994). Temporal control in fixed-interval schedules. *Journal of the Experimental Analysis of Behavior, 61*, 1-9.